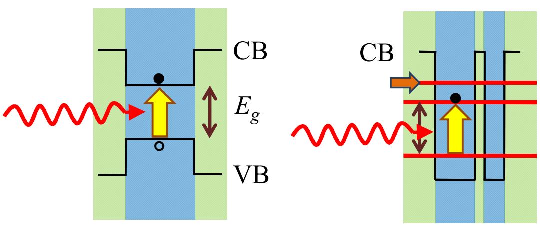
ELECTRON-PHOTON INTERACTION

Optical Absorption

- Transitions from valence band to conduction band
  → continuous range of absorption.
- Transitions between the levels of a quantum well
  → only discrete frequencies are permitted.



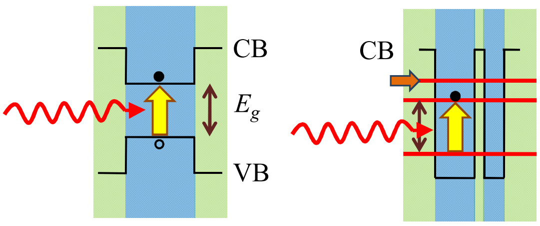
Light Wave

- Consider a light wave with an electric field:

$$\vec{E}\left(\vec{R},t\right) = 2\vec{e}E_0\cos\left(\vec{Q}\cdot\vec{R}-\omega t\right)$$

$\vec{e}$: Polarization of the electric field → $\vec{e}\perp\vec{Q}$

- A magnetic field exists, but the effect is negligible.



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Why Electric Force is Stronger

We will use a plane electro-magnetic wave approximation.

Lorentz Force Equation:

$$\vec{F} = \vec{F}_E + \vec{F}_B$$

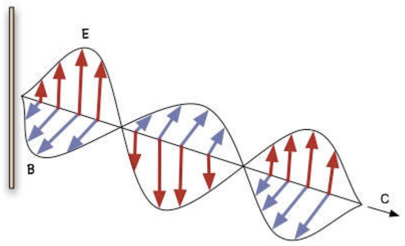
$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

$$E = E_0 e^{j(\omega t - kz)},\ \ B = B_0 e^{j(\omega t - kz)}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$kE_0 = \omega B_0,\ \ B_0 = \frac{k}{\omega}E_0 = \frac{E_0}{c}$$

$$\frac{F_B}{F_E} = \frac{v}{c} < 1$$



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Hamiltonian

- The Hamiltonian contains scalar and vector potentials.
- Vector potential:

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} \Rightarrow \vec{A}(R,t) = \frac{2\vec{e}E_0}{\omega}\sin\left(\vec{Q}\cdot\vec{R} - \omega t\right)$$

- Scalar potential:

$$\phi\left(\vec{R},t\right) \propto \mathrm{e}^{i\left(\vec{Q}\cdot\vec{R}-\omega t\right)} \Rightarrow E = -\nabla\phi \propto i\,\vec{Q}\,\mathrm{e}^{i\left(\vec{Q}\cdot\vec{R}-\omega t\right)}$$

→ Electric field is in the same direction as the direction of travel Q. This is in contrast to the transverse nature of electromagnetic wave.

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Perturbation

- Schrodinger equation for an electron:

$$\left[\frac{\left(\vec{p}+e\vec{A}\right)^2}{2m_0} + V_{\text{crystal}}\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

- The perturbation due to the light wave is the difference between this Hamiltonian and that without A.

$$\hat{V} = \frac{e}{2m_0}\left(\vec{A}\cdot\vec{p} + \vec{p}\cdot\vec{A} + e\vec{A}^2\right)$$

$\vec{A}^2$:

- Second order in the electric field and will be neglected.

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Perturbation

$$\hat{V} = \frac{e}{2m_0}\left(\vec{A}\cdot\hat{p} + \hat{p}\cdot\vec{A} + e\vec{A}^2\right)$$

$\hat{p}\cdot\vec{A}$:

$$\hat{p}\cdot\vec{A}\psi = -i\hbar\nabla\cdot\left(\vec{A}\psi\right) = -i\hbar\left[(\nabla\cdot\vec{A})\psi + \vec{A}\cdot(\nabla\psi)\right]$$

$\nabla\cdot\vec{A} = 0 \rightarrow$ Since the wave is transverse.

$$\hat{p}\cdot\vec{A}\psi = -i\hbar\vec{A}\cdot(\nabla\psi) = \vec{A}\cdot\hat{p}\psi$$

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Perturbation

$$\hat{V} = \frac{e}{m_0}\vec{A}\cdot\hat{p}$$

- Expanding perturbation

$$V(\vec{R},t) = \frac{eE_0}{im_0\omega}\left[e^{i(\vec{Q}\cdot\vec{R}-\omega t)} - e^{-i(\vec{Q}\cdot\vec{R}-\omega t)}\right](\vec{e}\cdot\hat{p})$$

- Absorption of a photon by the electron.
- Electron's energy increases by $\hbar\omega$.
- Electrons momentum increases by $\hbar Q$.

- Emission of a photon by the electron.
- Electron's energy decreases by $\hbar\omega$.
- Electron's momentum decreases by $\hbar Q$.

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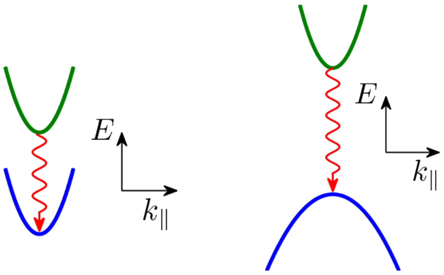
Electric-Dipole Approximation

- Think about light of visible or nearby wavelengths

  → Photon carries negligible momentum and induces vertical transitions in bands.

$$\vec{Q} = 0$$

$$V(\vec{R},t) = \frac{eE_0}{im_0\omega}\left[e^{-i\omega t} - e^{i\omega t}\right](\vec{e}\cdot\hat{p})$$



Fermi's Golden Rule

- The transition rate from a state *i* to another *j* due to the absorption of a photon

$$W_{ji} = \frac{2\pi}{\hbar}\left(\frac{eE_0}{m_0\omega}\right)^2 \left|\langle j\,|\,\vec{e}\cdot\hat{p}\,|\,i\rangle\right|^2 \delta\left(E_j - E_i - \hbar\omega\right)$$

$$\vec{e}\cdot\hat{p} = -i\hbar\left(e_x\frac{\partial}{\partial x} + e_y\frac{\partial}{\partial y} + e_z\frac{\partial}{\partial z}\right)$$

- For *z*-polarized electric field

$$\vec{e} = (0,0,1)$$

$$\vec{e}\cdot\hat{p} = -i\hbar\frac{\partial}{\partial z}$$

Power Absorbed

- Power absorbed in one transition = $W_{ji}\,\hbar\omega$
- Total Power absorbed, P_+:

$$P_+ = \hbar\omega \sum_{i,j} W_{ji}$$

- We should include the Fermi distribution:

$$P_+ = \hbar\omega \sum_{i,j} W_{ji} f(E_i)\left[1 - f(E_j)\right]\delta(E_j - E_i - \hbar\omega)$$

$f(E_i)$: Initial state is filled

$1 - f(E_j)$: Final state is empty

- If we consider the spin degeneracy

$$P_+ = 2\hbar\omega \sum_{i,j} W_{ji} f(E_i)\left[1 - f(E_j)\right]\delta(E_j - E_i - \hbar\omega)$$

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Power Absorption and Emission

- Absorption:

$$P_+ = \frac{2\pi}{\hbar}\hbar\omega\left(\frac{eE_0}{m_0\omega}\right)^2 2\sum_{i,j}\left|\langle j \,|\, \vec{e}\cdot\hat{p} \rangle i\right|^2 \times f(E_i)\left[1 - f(E_j)\right]\delta(E_j - E_i - \hbar\omega)$$

- Emission:

$$P_- = -\frac{2\pi}{\hbar}\hbar\omega\left(\frac{eE_0}{m_0\omega}\right)^2 2\sum_{i,j}\left|\langle j \,|\, \vec{e}\cdot\hat{p} \rangle i\right|^2 \times f(E_i)\left[1 - f(E_j)\right]\delta(E_j - E_i + \hbar\omega)$$

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Net Rate of Absorption of Energy

$$f(E_i)\left[1-f(E_j)\right]-f(E_j)\left[1-f(E_i)\right]=f(E_i)-f(E_j)$$

$$P=\frac{2\pi}{\hbar}\hbar\omega\left(\frac{eE_0}{m_0\omega}\right)^2 2\sum_{i,j}\left|\langle j\,|\,\vec{e}\cdot\hat{p}\rangle i\right|^2$$
$$\times\left[f(E_i)-f(E_j)\right]\delta\left(E_j-E_i-\hbar\omega\right)$$