ELECTRIC POTENTIAL

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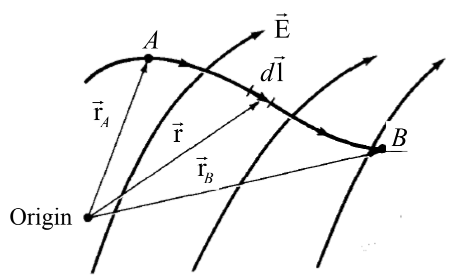
Electric Potential

- We obtained electric field from Coulomb's law and Gauss's law.
- Electric field can also be found from electric scalar potential *V*.
- Work done on moving *q* by $d\vec{l}$

$$dW = -\vec{F}\cdot d\vec{l} = -q\vec{E}\cdot d\vec{l}$$

$$W = -q\int_A^B \vec{E}\cdot d\vec{l}$$

$$V_{AB} = \frac{W}{q} = -\int_A^B \vec{E}\cdot d\vec{l}$$



Unit: Joules per coulomb or volt

V_{AB} is independent of the path!

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Potential Due to Point Charge

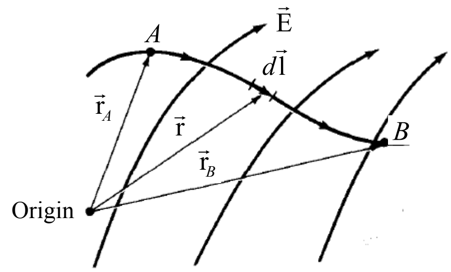
If a point charge Q is located at origin

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \qquad d\vec{l} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \cdot \left(dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi\right)$$

$$= -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \cdot dr\,\hat{a}_r$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$



$$V_{AB} = V_B - V_A$$

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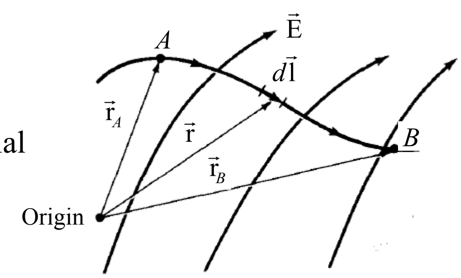
Potential Due to Point Charge

- If $V_A = 0$ as $r_A \to \infty$, $V = \dfrac{Q}{4\pi\varepsilon_0 r}$
- If Q is not located at the origin

$$V(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 |\vec{r} - \vec{r}'|}$$

- For an arbitrary reference potential

$$V(\vec{r}) = \frac{Q}{4\pi\varepsilon_0 r} + C$$



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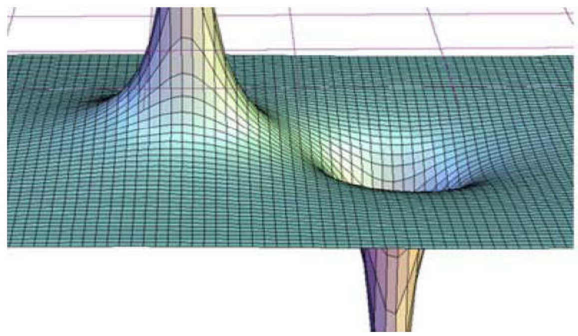
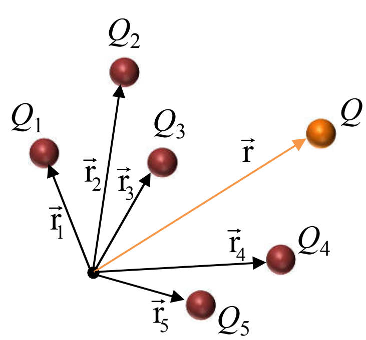
Potential Landscape

Positive charge



Negative charge

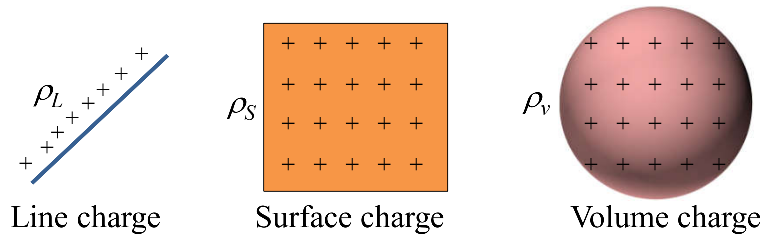
Superposition



$$V(\vec{r}) = \frac{Q_1}{4\pi\varepsilon_0 \,|\, \vec{r} - \vec{r}_1 \,|} + \frac{Q_2}{4\pi\varepsilon_0 \,|\, \vec{r} - \vec{r}_2 \,|} + \cdots + \frac{Q_n}{4\pi\varepsilon_0 \,|\, \vec{r} - \vec{r}_n \,|}$$

$$= \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{n} \frac{Q_k}{|\, \vec{r} - \vec{r}_k \,|}$$

Continuous Charge Distribution



$$V(\vec{\mathrm{r}}) = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\rho_L(\vec{\mathrm{r}}')\,dl'}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|} \qquad \text{(line charge)}$$

$$V(\vec{\mathrm{r}}) = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_S(\vec{\mathrm{r}}')\,dS'}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|} \qquad \text{(surface charge)}$$

$$V(\vec{\mathrm{r}}) = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho_v(\vec{\mathrm{r}}')\,dv'}{|\vec{\mathrm{r}}-\vec{\mathrm{r}}'|} \qquad \text{(volume charge)}$$

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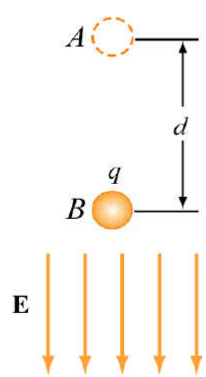
Potential in a Uniform Electric Field

$$\begin{aligned}\Delta V = V_B - V_A &= -\int_A^B \vec{\mathrm{E}}\cdot d\vec{\mathrm{l}} \\ &= -\int_A^B E\,\hat{\mathrm{a}}_z\cdot\hat{\mathrm{a}}_z\,dz \\ &= -\int_A^B E\,dz \\ &= -Ed\end{aligned}$$



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Second Maxwell's Equation

- V_{AB} is independent of the path.

$$V_{BA} = -V_{AB}$$
$$V_{BA} + V_{AB} = 0$$

- Line integral of $\vec{E}$ along a closed path is zero.

$$\oint \vec{E} \cdot d\vec{l} = 0$$

- Applying Stoke's theorem

$$\oint \vec{E} \cdot d\vec{l} = \int \left(\nabla \times \vec{E}\right) \cdot d\vec{S} = 0$$

$$\boxed{\nabla \times \vec{E} = 0}$$ → Second Maxwell's equation for static field

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Relation Between $\vec{E}$ and V

$$dV = -\vec{E} \cdot d\vec{l} = -E_x\,dx - E_y\,dy - E_z\,dz$$

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z},$$

$$\vec{E} = E_x\,\hat{a}_x + E_y\,\hat{a}_x + E_z\,\hat{a}_z$$

$$\vec{E} = -\left(\hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + \hat{a}_z \frac{\partial}{\partial z}\right)V$$

$$\boxed{\vec{E} = -\nabla V}$$

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Concept Question: Positive Charge

Place a positive charge in an electric field. It will accelerate from

1. higher to lower electric potential; lower to higher potential energy
2. higher to lower electric potential; higher to lower potential energy
3. lower to higher electric potential; lower to higher potential energy
4. lower to higher electric potential; higher to lower potential energy

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Concept Question: Negative Charge

Place a negative charge in an electric field. It will accelerate from

1. higher to lower electric potential; lower to higher potential energy
2. higher to lower electric potential; higher to lower potential energy
3. lower to higher electric potential; lower to higher potential energy
4. lower to higher electric potential; higher to lower potential energy

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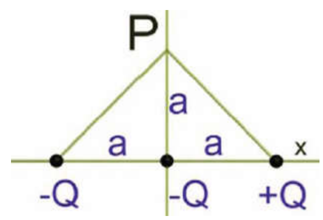
Concept Question: Superposition

Consider the point charges in the figure. What will be the potential at point P?

1. $V = -\frac{Q}{4\pi\varepsilon_0 a}$
2. $V = \frac{Q}{4\pi\varepsilon_0 a}$
3. $V = \frac{Q}{4\pi\varepsilon_0 a^2}$
4. 0



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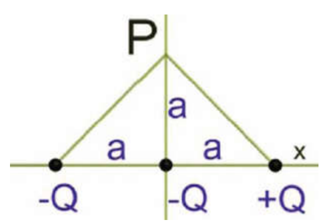
Concept Question: E from V

Consider the point charges in the figure. What is the electric field at P?

1. $Q/(4\pi\varepsilon_0 a^2)$
2. $-Q/(4\pi\varepsilon_0 a^2)$
3. 0
4. None of the above.



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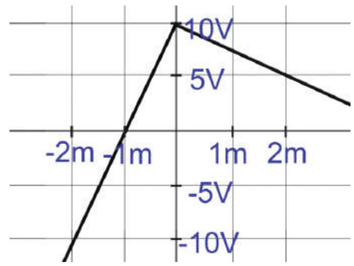
Concept Question: E from V

The graph shows a potential V as a function of x. The magnitude of the electric field for $x > 0$ is

1. larger than that for $x < 0$
2. smaller than that for $x < 0$
3. equal to that for $x < 0$
4. zero.



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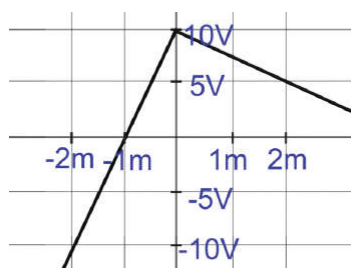
Concept Question: E from V

The graph shows a potential V as a function of x. Which is true?

1. $E_{x>0}$ is > 0 and $E_{x<0}$ is > 0,
2. $E_{x>0}$ is > 0 and $E_{x<0}$ is < 0,
3. $E_{x>0}$ is < 0 and $E_{x<0}$ is < 0,
4. $E_{x>0}$ is < 0 and $E_{x<0}$ is > 0.



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