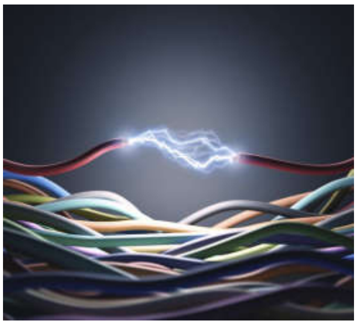
ELECTRICAL CONDUCTION



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Effect of Alloying on Resistivity

$$\rho = \frac{1}{en\mu_d} = \frac{1}{en\mu_L} + \frac{1}{en\mu_I} = \rho_T + \rho_I$$

In a binary alloy that forms a solid solution, we would expect the above equation to apply, with the temperature-independent impurity contribution ρ_I increasing with the concentration of solute atoms. This means that as the alloy concentration increases, the resistivity ρ increases and becomes less temperature dependent as ρ_I overwhelms ρ_T, leading to $\alpha \ll 1/273$.

Material	Resistivity at 20 ^oC (nΩ m)	α at 20 ^oC (1/K)
Nickel	69	0.0064
Chrome	129	0.0030
Nichrome (80%Ni-20% Cr)	1100	0.0004

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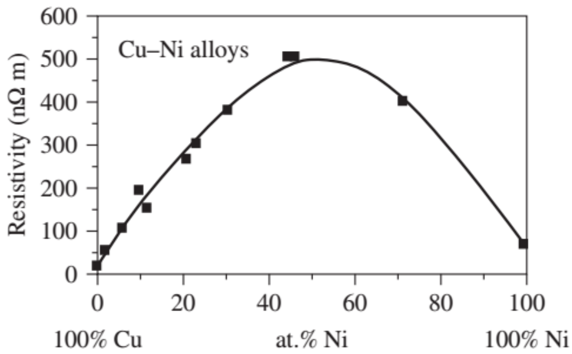
Effect of Alloying on Resistivity

- ρ of Cu–Ni alloy as a function of Ni content (at.%) at room temperature.



- **Nichrome** is widely used as a heater wire in household appliances and industrial furnaces

Nordheim's Rule

- Relates the impurity resistivity ρ_I to the atomic fraction X of solute atoms in a solid solution, as follows:

$$\rho_I = CX(1 - X)$$

where C is the constant termed the Nordheim coefficient, which represents the effectiveness of the solute atom in increasing the resistivity.

- For sufficiently small amounts of impurity, experiments show that the increase in the resistivity ρ_I is nearly always simply proportional to the impurity concentration X, that is, $\rho_I \propto X$.

Heterogenous Mixture

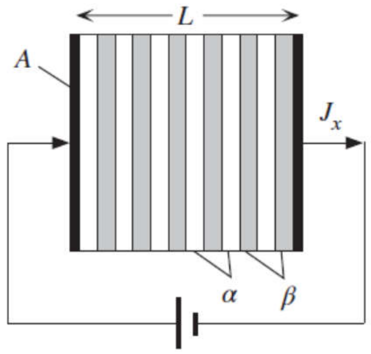
- Consider a material with two distinct phases α and β stacked in layers.
- Effective resistivity (R_{eff}) for current flow in the x direction?
- α and β layers are in parallel → series rule of mixtures → resistivity mixture rule

$$R_{\text{eff}} = \frac{L_\alpha \rho_\alpha}{A} + \frac{L_\beta \rho_\beta}{A}$$

$$R_{\text{eff}} = \frac{L \rho_{\text{eff}}}{A}$$

$$\rho_{\text{eff}} = \chi_a \rho_a + \chi_\beta \rho_\beta$$

$$\chi_a = \frac{L_a}{L} \qquad \chi_\beta = \frac{L_\beta}{L}$$



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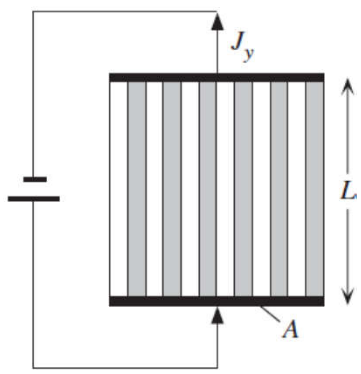
Heterogenous Mixture

- R_{eff} in the y-direction?
- α and β layers are in parallel → parallel rule of mixtures → conductivity mixture rule.

$$\sigma_{\text{eff}} = \chi_\alpha \sigma_\alpha + \chi_\beta \sigma_\beta$$



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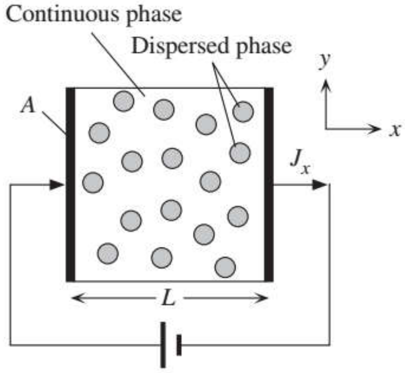
Heterogenous Mixture

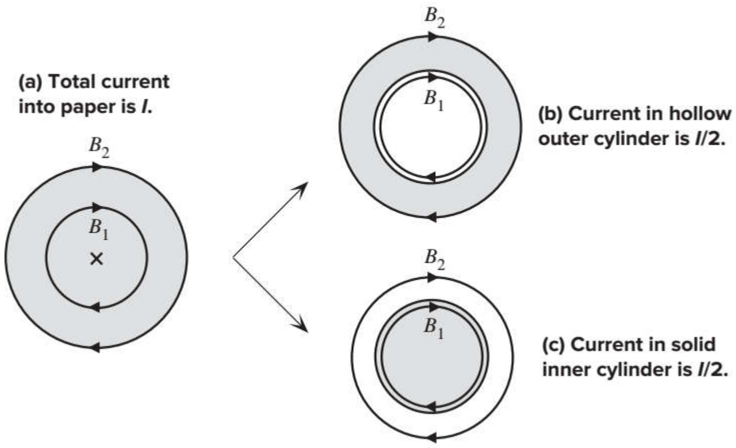
- R_{eff} for a random mixture of phase α and phase β?
- Series mixture rule → when ρc and ρd are not markedly different
- If the resistivity of one phase is appreciably different →
- **Empirical relations**

$$\rho_{\text{eff}} = \rho_c \frac{(1 + \frac{1}{2}\chi_d)}{(1 - \chi_d)} \qquad (\rho_d > 10\rho_c)$$

$$\rho_{\text{eff}} = \rho_c \frac{(1 - \chi_d)}{(1 + 2\chi_d)} \qquad (\rho_d < 0.1\rho_c)$$



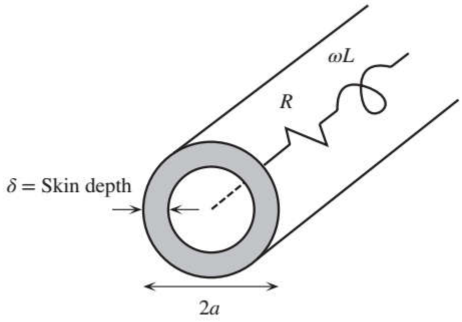
HF Resistance of a Conductor



Skin Depth

- For a given conductor, we can assume that most of the current flows in a surface region of depth δ, called the skin depth.
- In the central region, the current will be negligibly small.
- We can imagine the central conductor as a resistance R in series with an inductance L.

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$$



AC Resistance

- Effective cross-sectional area: $A = \pi a^2 - \pi(a-\delta)^2 \approx 2\pi a\delta$
- AC resistance: $r_{ac} = \frac{\rho}{A} \approx \frac{\rho}{2\pi a\delta}$
- The skin effect limits the use of solid-core conductors in high-frequency applications → frequencies > 10^9 Hz range, the transmission of the signal over a long distance becomes almost impossible through an ordinary, solid metal conductor. We must then resort to pipes (or waveguides).