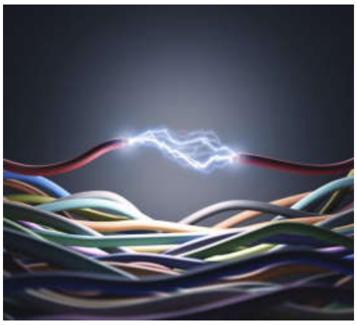
ELECTRICAL CONDUCTION



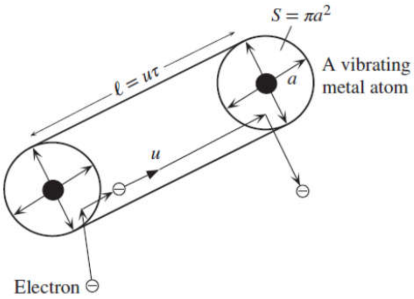
Mean Free Time

- Conduction electrons are scattered from the thermal vibrations of the atoms.
- $N_s(Su\tau) = 1 \rightarrow$ One scatterer.
- The mean free time between collisions:

$$\tau = \frac{1}{N_s S u}$$

- u → slightly depends on temperature.
- The temperature dependence of τ essentially arises from $S = \pi a^2$.

$$\tau \propto \frac{1}{\pi a^2}$$



Temperature Dependence of Resistivity

- Thermal vibrations of atoms can be considered to be simple harmonic motion, like a mass *M* attached to a spring.

$$\mathrm{K.E._{avg}} = \frac{1}{T}\int_0^T \frac{1}{2} M v^2 dt = \frac{1}{T}\int_0^T \frac{1}{2} M a^2 \omega^2 \cos^2 \omega t dt$$

$$\mathrm{K.E._{avg}} = \frac{1}{4} M a^2 \omega^2$$

- Kinetic molecular theory → $\mathrm{K.E._{avg}} = \frac{1}{2} kT$

$$\frac{1}{4} M a^2 \omega^2 \approx \frac{1}{2} kT \rightarrow a^2 \propto T$$

$$\tau \propto \frac{1}{\pi a^2} \propto \frac{1}{T} \rightarrow \tau = \frac{C}{T}$$

Lattice-Scattering-Limited Conductivity

$$\mu_d = \frac{e\tau}{m_e} = \frac{eC}{m_e T}$$

So, the resistivity ρ_T of a pure metal is

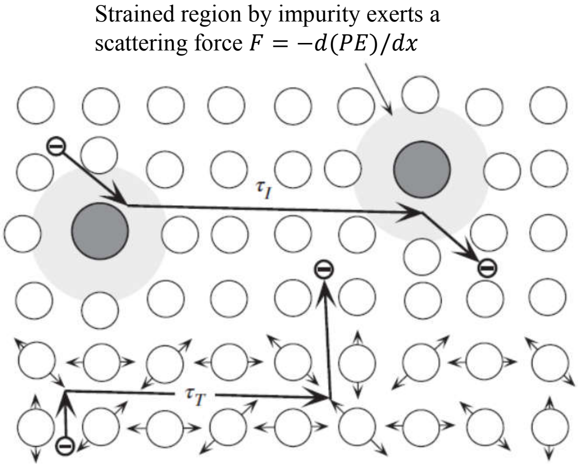
$$\rho_T = \frac{1}{\sigma_T} = \frac{1}{en\mu_d} = \frac{m_e T}{e^2 n C}$$

$\rho_T = AT$ → lattice-scattering-limited conductivity

Matthiessen's Rule

- $\rho_T = AT$ → works well with pure metals

  → Fails for metallic alloys.


Strained region by impurity exerts a scattering force $F = -d(PE)/dx$

Effective Mean Free Time

- Two different types of scattering processes: (1) from impurities alone and (2) from thermal vibrations alone.
- Two types of mean free times between collisions: (1) τ_T for scattering from thermal vibrations only, and (2) τ_I for scattering from impurities only.

$$\frac{1}{\tau} = \frac{1}{\tau_T} + \frac{1}{\tau_I}$$ → τ is smaller than both τ_T and τ_I.

$$\frac{1}{\mu_d} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

μ_d: drift mobility
μ_L: lattice-scattering-limited mobility
μ_I: impurity-scattering-limited mobility

$$\rho = \frac{1}{en\mu_d} = \frac{1}{en\mu_L} + \frac{1}{en\mu_I} = \rho_T + \rho_I$$

Residual Resistivity

There may also be electrons scattering from dislocations and other crystal defects, as well as from grain boundaries. All of these scattering processes add to the resistivity of a metal, just as the scattering process from impurities. We can therefore write the effective resistivity of a metal as

$$\rho = \rho_T + \rho_R$$

ρ_R : residual resistivity due to the scattering of electrons by impurities, dislocations, interstitial atoms, vacancies, grain boundaries, etc.

Residual resistivity shows very little temperature dependence.

$$\rho \approx AT + B$$

where A and B are temperature-independent constants.

Temperature Coefficient

- The temperature coefficient of resistivity (TCR) α_0 is defined as the fractional change in the resistivity per unit temperature increase at the reference temperature T_0

$$\alpha_0 = \frac{1}{\rho_0}\left[\frac{\delta\rho}{\delta T}\right]_{T=T_0}$$

- T_0 → usually 273 K (0 °C) or 293 K (20 °C)

$$\delta\rho = \rho - \rho_0$$

$$\delta T = T - T_0$$

when α_0 is constant over the temperature range of interest

$$\rho = \rho_0[1 + \alpha_0 (T - T_0)]$$

- **For metals:** $\alpha_0 = \frac{1}{T_0}$ → For $T_0 = 273$ K, $\alpha_0 = 1/273$.

α_0 at 273 K

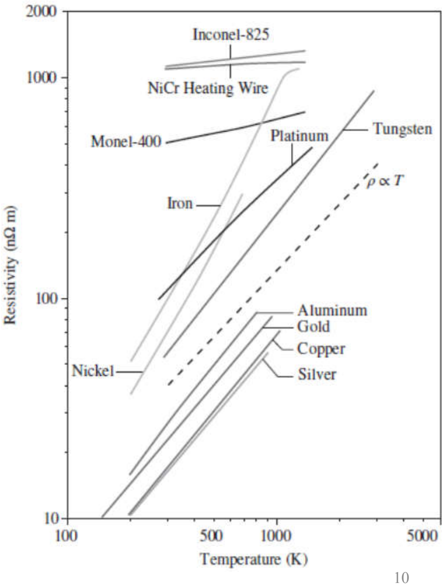
Metal	α_0
Aluminum, Al	1/233
Antimony, Sb	1/196
Copper, Cu	1/232
Gold, Au	1/251
Indium, In	1/196
Platinum, Pt	1/255
Silver, Ag	1/244
Tantalum, Ta	1/294
Tin, Sn	1/217
Tungsten, W	1/202
Iron, Fe	1/152
Nickel, Ni	1/125

Resistivity vs. Temperature

- $\rho \propto T$ is approximately obeyed *except for the magnetic materials.*
- For alloys, such as nichrome (Ni-Cr), ρ is relatively temperature insensitive, with a very small TCR.
- **Empirical relation** between ρ and T for pure metals:

$$\rho = \rho_0 \left[\frac{T}{T_0}\right]^n$$

- For nonmagnetic metals, $n \approx 1$, whereas it is closer to 2 than 1 for the magnetic metals Fe and Ni.



Resistivity vs. Temperature

$\rho = AT + B$

$T \lesssim 100$K:

- The number of atoms that vibrate with sufficient energy to scatter the conduction electrons starts to decrease rapidly with decreasing temperature → ρ becomes more strongly temperature dependent

  $\rightarrow \rho \propto T^5$

- $\rho = DT^5 + \rho_R$

- At **$T \rightarrow 0$**, ρ is limited by scattering from impurities and crystal defects.