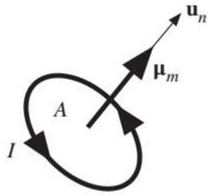
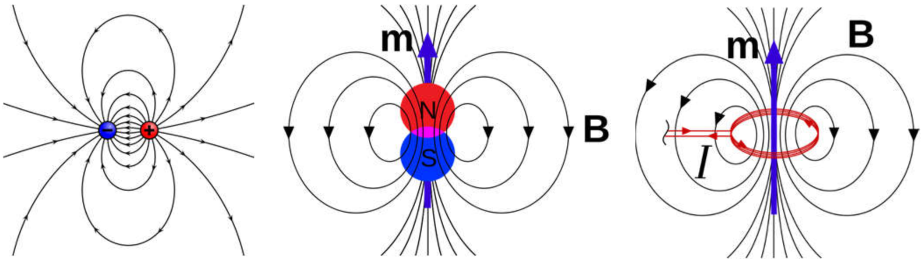
MAGNETIZATION

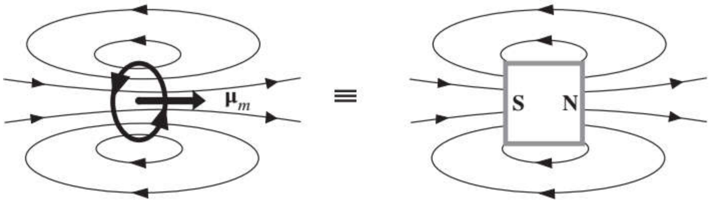
Magnetic Dipole Moment

- The **magnetic moment** is the magnetic strength and orientation of a magnet or other object that produces a magnetic field.
- **Examples** of objects that have magnetic moments include: loops of electric current (such as electromagnets), permanent magnets, elementary particles (such as electrons), various molecules, and many astronomical objects (such as many planets, some moons, stars, etc).
- Consider a current loop: $\boldsymbol{\mu}_m = IA\mathbf{u}_n$



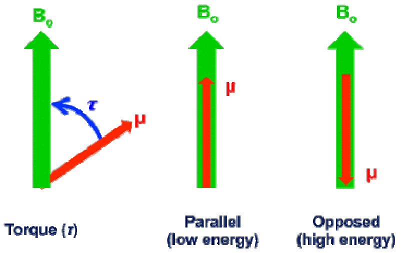
Magnetic Dipole

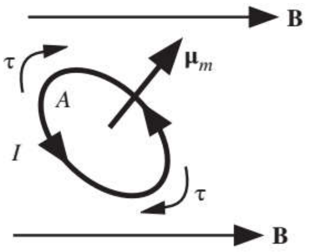




In External Field

- A magnetic field, applies a rotating torque on the magnetic moment to align its axis with the magnetic field.
- The magnetic dipole moment is defined by the torque that object experiences.
- The strength (and direction) of this torque depends on both its magnitude and on orientation relative to the direction of the magnetic field.





Atomic Magnetic Moments

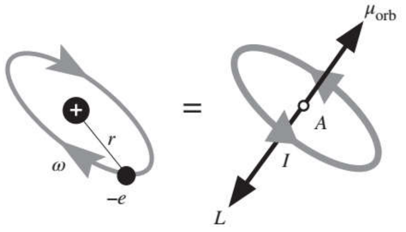
Orbital Magnetic Moment

$$I = \text{Charge flowing per unit time} = -\frac{e}{\text{Period}} = -\frac{e\omega}{2\pi}$$

$$\mu_{\text{orb}} = I(\pi r^2) = -\frac{e\omega r^2}{2}$$

$$L = m_e v r = m_e \omega r^2$$

$$\mu_{\text{orb}} = -\frac{e}{2m_e}L$$



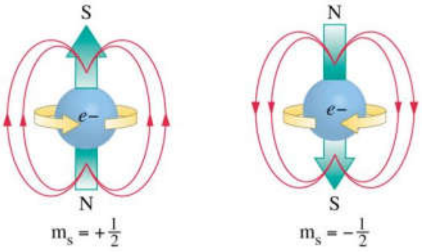
- Magnetic moment is proportional to the orbital angular momentum through the factor $e/2m_e$ → **gyromagnetic ratio.**
- The negative sign in Equation indicates that μ_{orb} is in the opposite direction to L and is due to the negative charge of the electron.

Atomic Magnetic Moments

Spin Magnetic Moment

- The electron also has an intrinsic angular momentum S, that is, spin. The spin of the electron has a **spin magnetic moment,** denoted by μ_{spin}.

$$\mu_{\text{spin}} = -\frac{e}{m_e}S$$

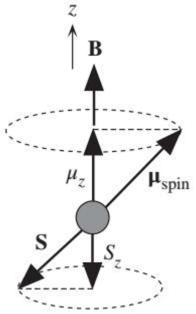


- The overall magnetic moment of the electron consists of $\boldsymbol{\mu}_{\text{orb}}$ and $\boldsymbol{\mu}_{\text{spin}}$ vectorially added.
- The overall magnetic moment $\boldsymbol{\mu}_{\text{atom}}$ depends on the orbital motions and spins of *all* the electrons.
- Electrons in closed subshells do not contribute to the overall magnetic moment.

Precession

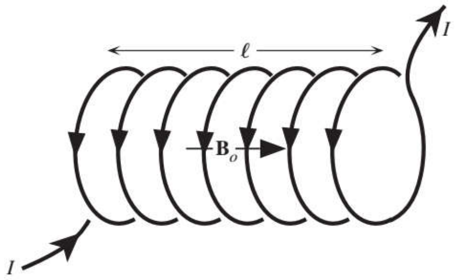
- Consider an atom that has closed inner shells and a single electron in an *s* orbital (ℓ = 0).
- The spin magnetic moment precesses about an external magnetic field along z and has a value μ_z along z.

$$\mu_z = -\frac{e}{m_e}S_z = -\frac{e}{m_e}m_s\hbar = \frac{e\hbar}{2m_e} = \beta$$



Magnetization

- Consider a tightly wound long solenoid, ideally infinitely long, with free space (or vacuum) as the medium inside the solenoid.

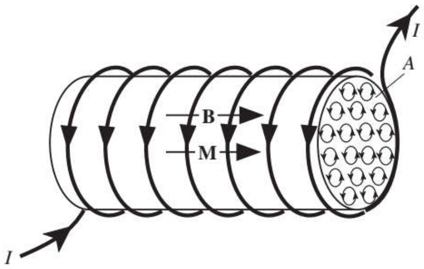


$$B_0 = \mu_0 nI = \mu_0 I'$$

I': Current per unit length
μ_0: Permeability of free space

Magnetization

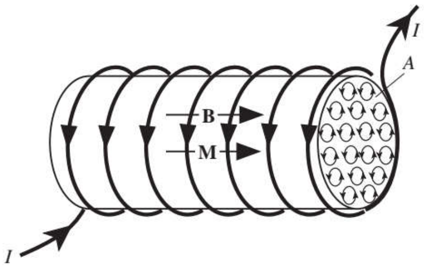
- Let us now place a cylindrical material to fill the inside of this solenoid.



- The magnetic field has changed from $\mathbf{B}_0$ to $\mathbf{B}$.
- $\mathbf{B}_0$ to be the applied magnetic field into which the material medium is placed.

Magnetic Dipole Moment

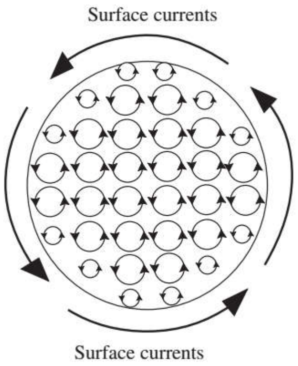
- Let us now place a cylindrical material to fill the inside of this solenoid.



- Each atom of the material responds to the applied field $\mathbf{B}_0$ and develops, or acquires, a net magnetic moment $\boldsymbol{\mu}_m$ along the applied field.
- **Magnetic dipole moment per unit volume (M):** If there are N atoms in a small volume ΔV and each atom i has a magnetic moment $\boldsymbol{\mu}_{mi}$ →

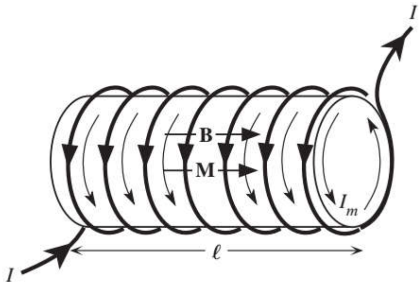
$$\mathbf{M} = \frac{1}{\Delta V}\sum_{i=1}^{N} \boldsymbol{\mu}_{mi} = n_{\mathrm{at}}\boldsymbol{\mu}_{\mathrm{av}}$$

Surface Currents



- All neighboring atomic current loops in the bulk have adjacent currents in opposite directions that cancel each other.
- However, the currents at the surface in the surface loops cannot be cancelled and this leads to a net **surface current**.
- Total $\mathbf{M} = M \times \text{Volume} = MAl$
- Total $\mathbf{M} = \text{Total current} \times \text{Cross-sectional area} = I_m lA$
- $M = I_m \rightarrow$ Surface current per unit length

Magnetic Field



$$B = \mu_0(I' + I_m) = B_0 + \mu_0 M$$

$$\mathbf{B} = \mathbf{B_0} + \mu_0\mathbf{M}$$

- Magnetizing field or magnetic field intensity (**H**):

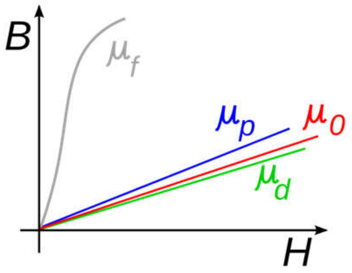
$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M} \qquad \mathbf{H} = \frac{\mathbf{B_0}}{\mu_0}$$

$H = nI =$ Total conduction current per unit length

Magnetic Permeability

- **Magnetic permeability:** Magnetic field (B) per unit magnetizing field (H)

$$\mu = \frac{B}{H}$$



- μ relates the effect B to the cause H at the same point inside a material.
- Qualitatively, μ represents to what extent a medium is permeable by magnetic fields.

- **Relative permeability:**

$$\mu_r = \frac{B}{B_0} = \frac{B}{\mu_0 H} \rightarrow \mu = \mu_0 \mu_r$$

Magnetic Susceptibility

- **Magnetic susceptibility (χ_m):**

$$\mathbf{M} = \chi_m \mathbf{H}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

$$B = \mu_0 H + \mu_0 M = \mu_0 H + \mu_0 \chi_m H = \mu_0(1 + \chi_m)H$$

$$\mu_r = 1 + \chi_m$$