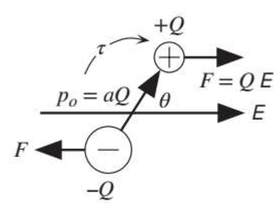
DIELECTRIC CONSTANT AND DIELECTRIC LOSS

1

Time-Varying Field

- When the applied field varies with time, the polarization is generally different than the static case.
- Let us consider orientational polarization involving dipolar molecules.
- The time-varying field changes magnitude and direction continuously, and it tries to line up the dipoles one way and then the other way and so on.



- If the instantaneous induced dipole moment p per molecule can instantaneously follow the field variations, then at any instant

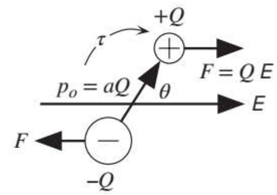
$$p = \alpha_d E$$

2

2

Alignment

- There are two factors opposing the immediate alignment of the dipoles with the field.
- **First:** Thermal agitation randomizes the dipole orientations
- **Second:** Interactions with neighbors.
- As a result, dipoles cannot respond instantaneously to the changes in the applied field.

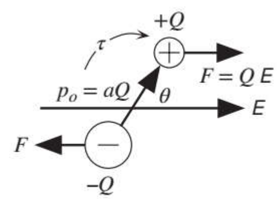


3

3

Polarizability vs. Frequency

- If the field changes too rapidly, then the dipoles cannot follow the field.
- At high frequencies, therefore, α_d will be zero as the field cannot induce a dipole moment.
- At low frequencies, of course, the dipoles can respond rapidly to follow the field and α_d has its maximum value.
- **We need to find the behavior of *αd* as a function of frequency *ω*.**



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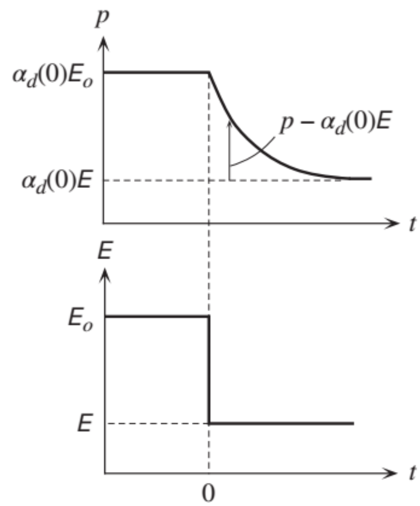
4

Change in Field

- Suppose that after a prolonged application, corresponding to dc conditions, the applied field across the dipolar gaseous medium is suddenly decreased from E_o to E at a time we define as zero,

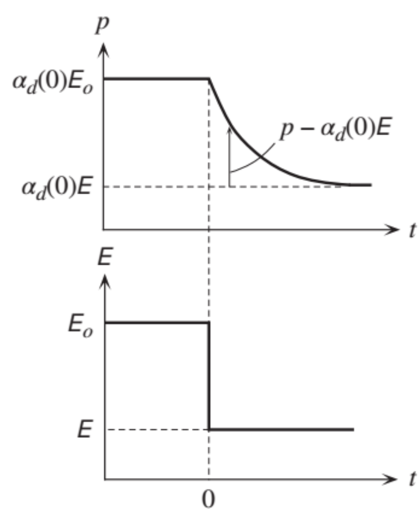


5

Relaxation

- **τ:** Average time , called the relaxation time, between molecular collisions, then this is the mean time it takes per molecule to randomize the induced dipole moment.

$$\frac{dp}{dt} = -\frac{p - \alpha_d(0)E}{\tau}$$



6

Sinusoidal Electric Field

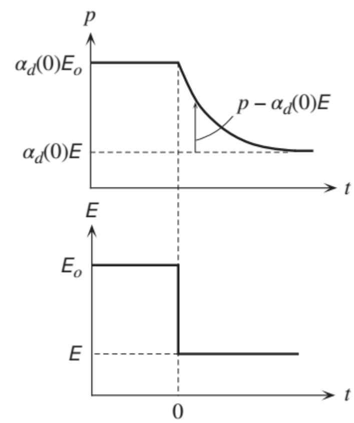
$$E = E_o \exp(j\omega t)$$

$$\frac{dp}{dt} = -\frac{p}{\tau} + \frac{\alpha_d(0)}{\tau} E_o \exp(j\omega t)$$

$$p = \alpha_d(\omega) E_o \exp(j\omega t)$$

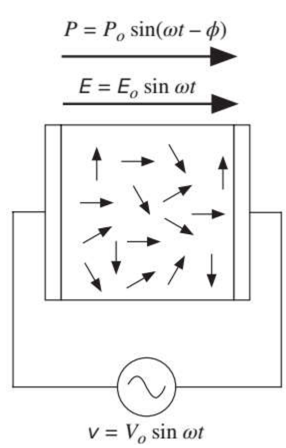
$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau}$$

- $\alpha_d(\omega)$ is a complex number that indicates that p and E are out of phase
- At low frequencies, $\omega\tau \ll 1$, $\alpha_d(\omega)$ is nearly $\alpha_d(0)$, and p is in phase with E.



Phase Delay

$$\alpha_d(\omega) = \frac{\alpha_d(0)}{1 + j\omega\tau}$$



Complex Dielectric Constant

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}$$

$$\varepsilon_r = 1 + \frac{N}{\varepsilon_0}\,\frac{\alpha_d(0)}{1 + j\omega\tau}$$

$$\varepsilon_r = 1 + \frac{N}{\varepsilon_0}\,\frac{\alpha_d(0)}{1 + j\omega\tau} \times \frac{1 - j\omega\tau}{1 - j\omega\tau}$$

$$\varepsilon_r = 1 + \frac{N}{\varepsilon_0}\,\frac{\alpha_d(0)}{1 + \omega^2\tau^2} - j\frac{N}{\varepsilon_0}\,\frac{\alpha_d(0)\omega\tau}{1 + \omega^2\tau^2}$$

$$\varepsilon_r = \varepsilon'_r - j\varepsilon''_r$$

9

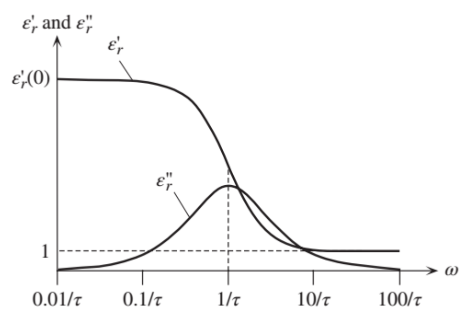
9

Frequency Dependence

$$\varepsilon_r = \varepsilon'_r - j\varepsilon''_r$$

$$\varepsilon'_r = 1 + \frac{N}{\varepsilon_0}\,\frac{\alpha_d(0)}{1 + \omega^2\tau^2}$$

$$\varepsilon''_r = \frac{N}{\varepsilon_0}\,\frac{\alpha_d(0)\omega\tau}{1 + \omega^2\tau^2}$$



- ε'_r decreases from its maximum value $\varepsilon'_r(0)$ to 1 at high frequencies.
- $\varepsilon''_r(\omega)$ is zero at low and high frequencies but peaks when $\omega\tau = 1$ or when $\omega = 1/\tau$.

10

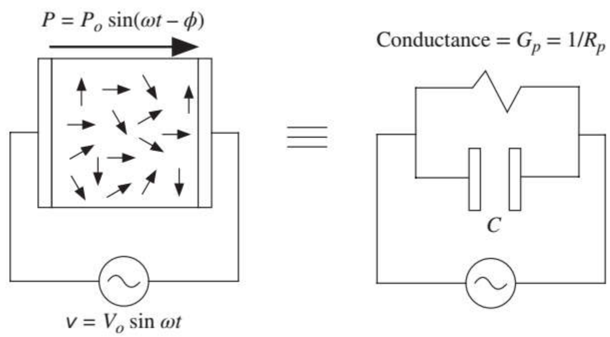
10

Lossless Capacitor & Parallel Conductance

- Admittance:

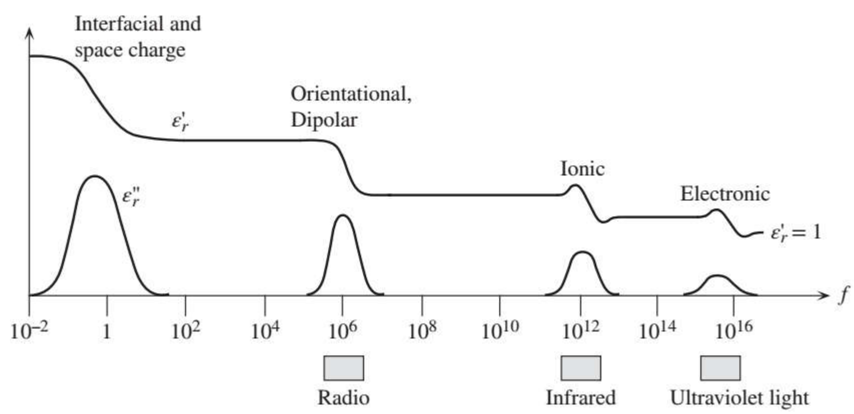
$$Y = \frac{j\omega A\varepsilon_o\varepsilon_r(\omega)}{d} = \frac{j\omega A\varepsilon_o\varepsilon_r'(\omega)}{d} + \frac{\omega A\varepsilon_o\varepsilon_r''(\omega)}{d} \rightarrow Y = j\omega C + G_P$$

$$C = \frac{A\varepsilon_o\varepsilon_r'}{d} \qquad G_P = \frac{\omega A\varepsilon_o\varepsilon_r''}{d}$$



11

Dielectric Resonance



When $\omega = 1/\tau$, energy is being transferred to heat most efficiently.

12