EFFECTIVE MASS

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Effective Mass

- A particle's effective mass (often denoted m^*) is the mass that it seems to have when responding to forces.
- The effective mass is usually stated in units of the true mass of the electron m_e (9.11×10^{-31} kg). In these units it is usually in the range 0.01 to 10, but can also be lower or higher.
- The electronic effective mass can be seen as an important basic parameter that influences measurable properties of a solid, including everything from the efficiency of a solar cell to the speed of an integrated circuit.

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Parabolic Isotropic Dispersion

- At the highest energies of the valence band in many semiconductors (Ge, Si, GaAs), and the lowest energies of the conduction band in some semiconductors (GaAs), the band structure $E(\mathbf{k})$ can be locally approximated as:

$$E\left(\vec{k}\right) = E_0 + \frac{\hbar^2 \vec{k}^2}{2m^*}$$

- It can be shown that the electrons placed in these bands behave as free electrons except with a different mass, as long as their energy stays within the range of validity of the approximation above. As a result, the electron mass in models such as the Drude model must be replaced with the effective mass.

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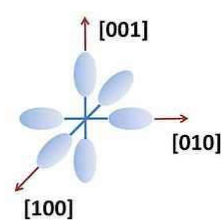
Parabolic Non-isotropic Dispersion

- In some important semiconductors (notably, silicon) the lowest energies of the conduction band are not symmetrical so that the band minimum can be approximated only by



$$E\left(k\right) = E_0 + \frac{\hbar^2}{2m_x^*}\left(k_x - k_{0,x}\right)^2 + \frac{\hbar^2}{2m_y^*}\left(k_y - k_{0,y}\right)^2 + \frac{\hbar^2}{2m_z^*}\left(k_z - k_{0,z}\right)^2$$

- The speed of an electron will depend on its direction, and it will accelerate to a different degree depending on the direction of the force.
- For the purposes of calculating conductivity as in the Drude model, via the harmonic mean

$$m^*_{\text{conductivity}} = 3\left[\frac{1}{m_x^*} + \frac{1}{m_y^*} + \frac{1}{m_z^*}\right]^{-1}$$

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General Case

- The dispersion relation:

$$\varepsilon = \hbar\omega$$

- The electron's acceleration:

$$\frac{dv_g}{dt} = \frac{d}{dt}\left(\frac{d\omega}{dk}\right) = \frac{d}{dk}\left(\frac{d\omega}{dk}\right)\frac{dk}{dt} = \frac{1}{\hbar^2}\frac{d^2\varepsilon}{dk^2}\frac{d(\hbar k)}{dt}$$

- If force F_e accelerates electron, will do work dW_e in time dt, while electron's energy and momentum change by $d\varepsilon$ and dk.

$$dW_e = F_e dx = F_e v_g dt = F_e \frac{d\omega}{dk} dt$$

$$dW_e = d\varepsilon = \frac{d\varepsilon}{dk} dk = \hbar \frac{d\omega}{dk} dk$$

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Effective Mass

- Equating: $$F_e = \frac{d(\hbar k)}{dt}$$

- After substituting: $$\frac{dv_g}{dt} = \frac{1}{\hbar^2}\frac{d^2\varepsilon}{dk^2}F_e = \frac{F_e}{m^*}$$

$$m^* = \frac{\hbar^2}{d^2\varepsilon / dk^2}$$ → Effective mass

Material	Electron Effective Mass	Hole Effective Mass
Group IV		
Si (4.2 K)	1.08	0.56
Ge	0.555	0.37
Groups III-IV		
GaAs	0.067	0.45
InSb	0.013	0.60
Groups II-VI		
ZnO	0.,19	1.21
ZnSe	0.17	1.44

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Free Electron

$$\varepsilon = \frac{\hbar^2 k^2}{2m} \Rightarrow \frac{d^2\varepsilon}{dk^2} = \frac{\hbar^2}{m}$$

- Effective mass:

$$m^* = \frac{\hbar^2}{\hbar^2 / m} = m$$

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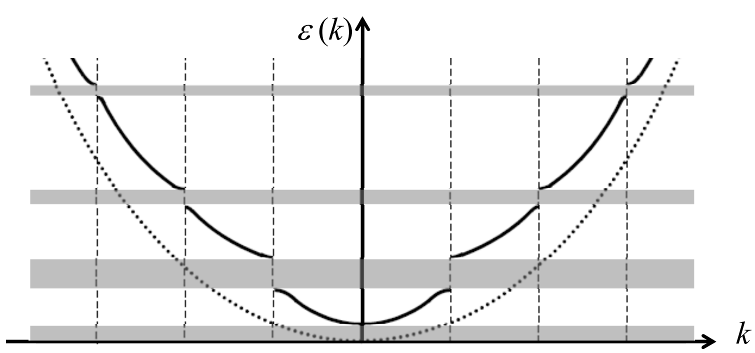
Effective Mass

- In parabolic sections: Near the top and bottom of the bands:

$$\varepsilon = C(k - n\pi)^2 \Rightarrow \frac{d^2\varepsilon}{dk^2} = 2C \Rightarrow m^* = \frac{\hbar^2}{2C}$$

- In non-parabolic sections: $d^2\varepsilon / dk^2$ depends on energy $\rightarrow m^*$ is not constant.



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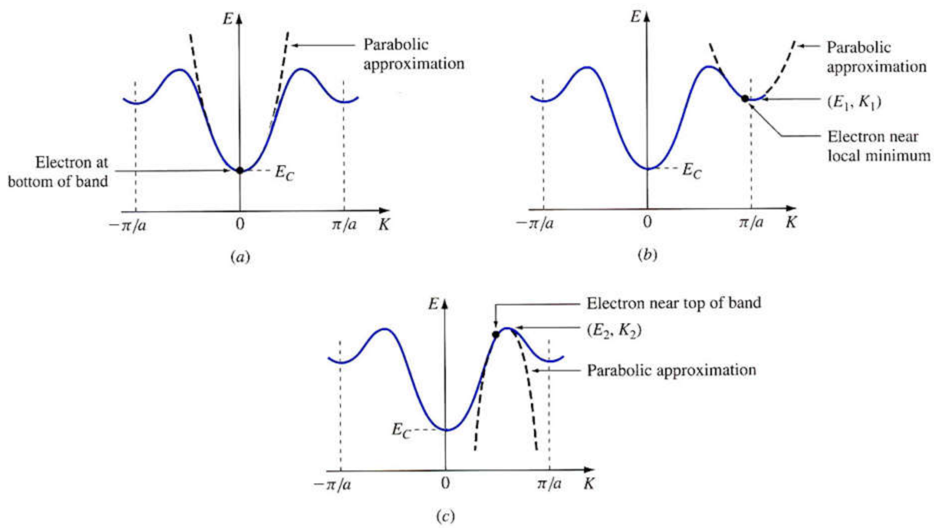
Parabolic Approximation



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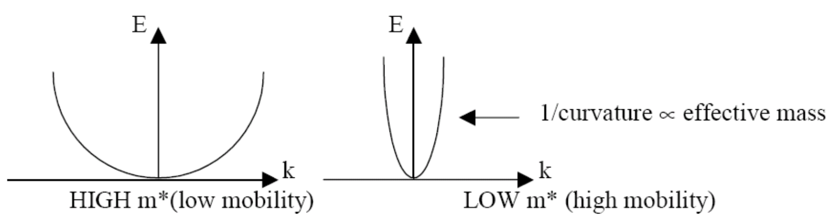
Effective Mass



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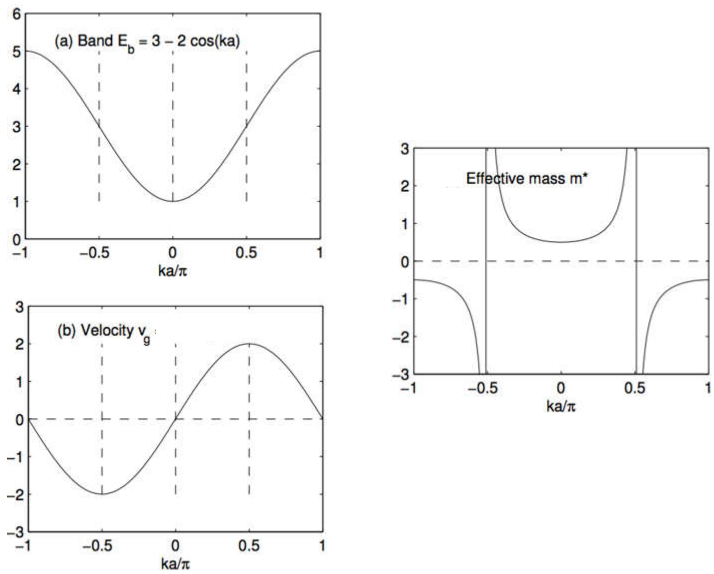
Effective Mass



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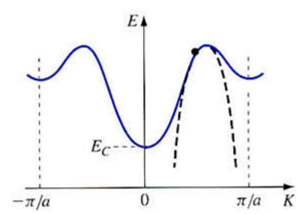
Negative Effective Mass

- Negative curvatures near the top of dispersion relation → negative effective mass.
- External field *E* in the positive *x*-direction acting on a negatively charged particle with a negative mass produces a positive acceleration component along the *x*-direction.
- The change in sign of *m** can be thought of the change of sign of charge *q*.



$$\frac{dv_g}{dt} = \frac{qE}{m^*}$$

- Quasi-particle → positive charge and positive mass → holes.
- Without lattice, holes cannot exist.

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