TUNNELING

1

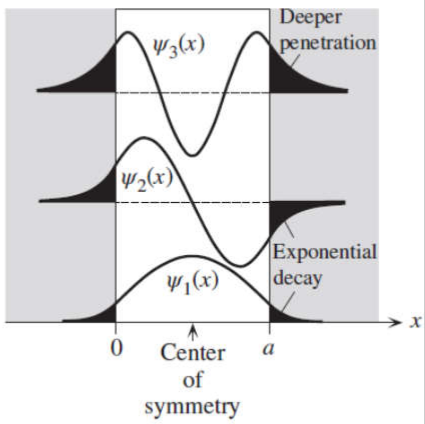
Bound Solutions

$$\psi(x)=\begin{cases} A_1 e^{\alpha x} & \text{for } x \le 0 \\ B_1 \cos kx + B_2 \sin kx & \text{for } 0 \le x \le a \\ C_2 e^{-\alpha x} & \text{for } x \ge a \end{cases}$$

$$\alpha=\sqrt{\frac{2m_e\left(V_0-E\right)}{\hbar^2}}$$

$$k=\sqrt{\frac{2mE}{\hbar^2}}$$

- **Penetration depth (δ):** The quantity $1/\alpha$ is a measure of the extent of penetration of the electron into the barrier.



Penetration Depth

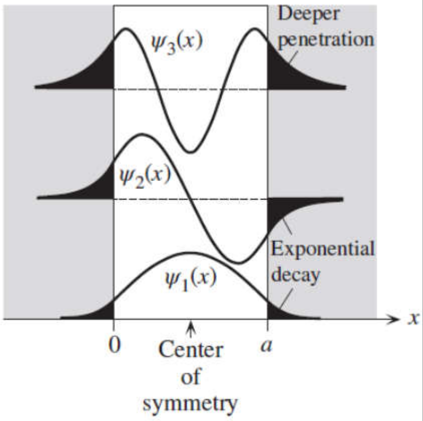
$$\alpha = \sqrt{\frac{2m_e(V_0 - E)}{\hbar^2}}$$

$$\delta = \frac{1}{\alpha}$$

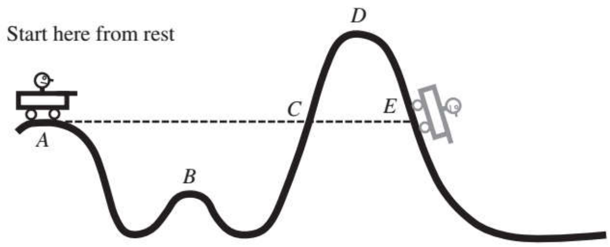
$a = 2$ nm, $V_0 = 0.5$ eV

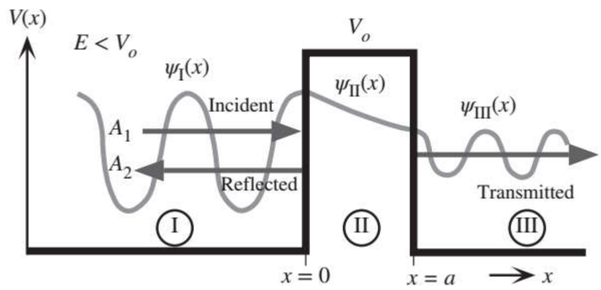
$E_1 = 0.057$ eV
$E_2 = 0.22$ eV
$E_3 = 0.45$ eV

$\delta_1 = 0.29$ nm
$\delta_2 = 0.37$ nm
$\delta_3 = 0.87$ nm



Classical vs. Quantum Mechanical





Electronic Wavefunctions

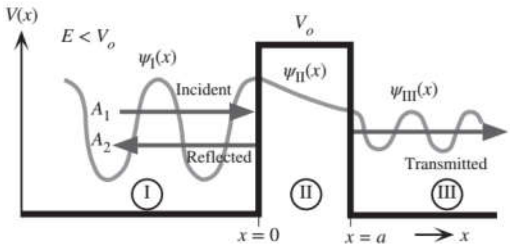
- We can divide the electron's space into three regions, I, II, and III and solve the Schrödinger equation for each region.

$$\psi_{\mathrm{I}}(x) = A_1 \exp(jkx) + A_2 \exp(-jkx)$$
$$\psi_{\mathrm{II}}(x) = B_1 \exp(\alpha x) + B_2 \exp(-\alpha x)$$
$$\psi_{\mathrm{III}}(x) = C_1 \exp(jkx) + C_2 \exp(-jkx)$$

$$k^2 = \frac{2m_e E}{\hbar^2}$$

$$\alpha^2 = \frac{2m_e(V_0 - E)}{\hbar^2}$$

- **Region I:** $A_1 \exp(-jkx) \rightarrow$ incident wave in $+x$, $A_2 \exp(jkx) \rightarrow$ reflected wave in $-x$ direction.
- **Region III:** $C_2 = 0 \rightarrow$ no reflected wave.

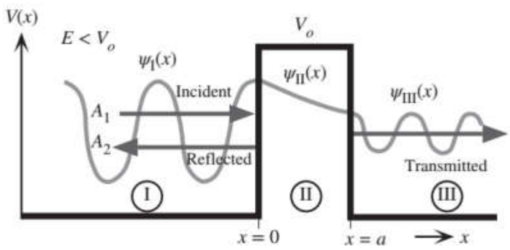


Tunneling Coefficient

- A_1, A_2, B_1, B_2, and C_1 are determined by applying the boundary conditions and the normalization condition.
- **Transmission coefficient (T):** The relative probability that the electron will tunnel from region I through II to III.

$$T = \frac{|\psi_{\mathrm{III}}(x)|^2}{|\psi_{\mathrm{I}}(\text{incident})|^2} = \frac{C_1^2}{A_1^2} = \frac{1}{1 + D \sinh^2(\alpha a)}$$

$$D = \frac{V_0^2}{4E(V_0 - E)}$$



Tunneling Coefficient

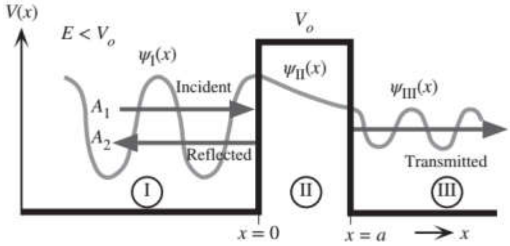
- For a wide or high barrier, using $\alpha a \gg 1$ and $\sinh(\alpha a) \approx \exp(\alpha a)/2$,

$$T = T_0 \exp(-2\alpha a) \qquad T_0 = \frac{16E(V_0 - E)}{V_0^2}$$

- Reflection coefficient (R):

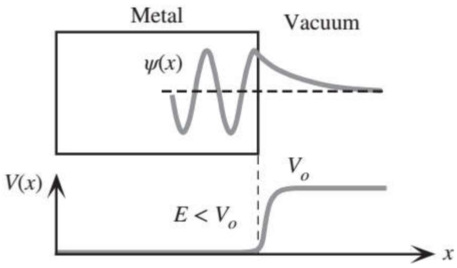
$$R = \frac{A_2^2}{A_1^2} = 1 - T$$

- The wider or higher the potential barrier, the smaller the chance of the electron tunneling.

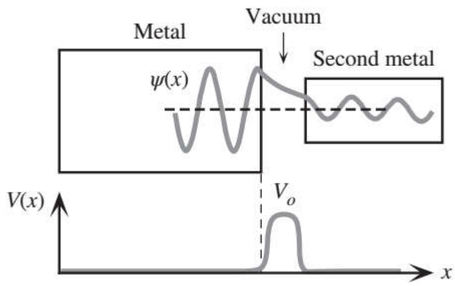


Tunneling

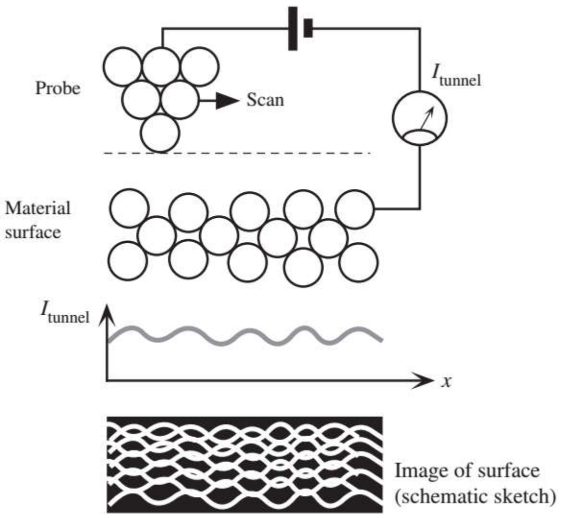
- The wavefunction decays exponentially as we move away from the surface because the PE outside the metal is Vo and the energy of the electron, E < Vo.

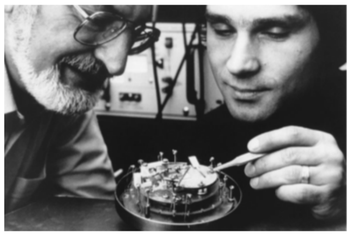


- If we bring a second metal close to the first metal, then the wavefunction can penetrate into the second metal → electron tunneling.



Scanning Tunneling Microscope

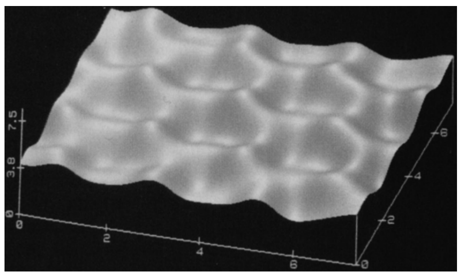


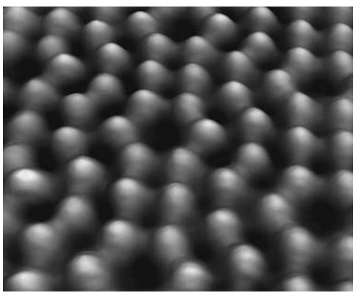


STM was invented by Gerd Binning (right) and Heinrich Rohrer (left) at the IBM Research Laboratory in Zurich, for which they were awarded the **1986 Nobel prize**.

STM Images

- STIM image of graphite surface: Contours represent electron concentrations within the surface. Carbon rings are clearly visible. The scale is in 2 Å.





- STM image of a Ni (110) surface.