# FINITE POTENTIAL ENERGY WELL

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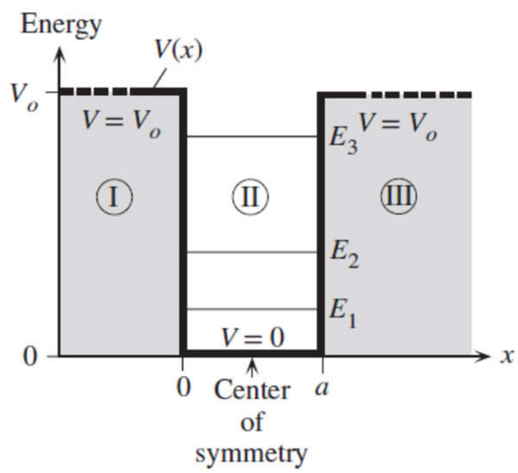
## *Finite Potential Well*

- A finite potential energy well has zero potential energy ($V = 0$) inside the well ($0 \le x \le a$) but a finite potential energy ($V = V_0$) outside the well ($x < 0$ and $x > a$).

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# Classically Allowed Region

- **$E > V_0$:**

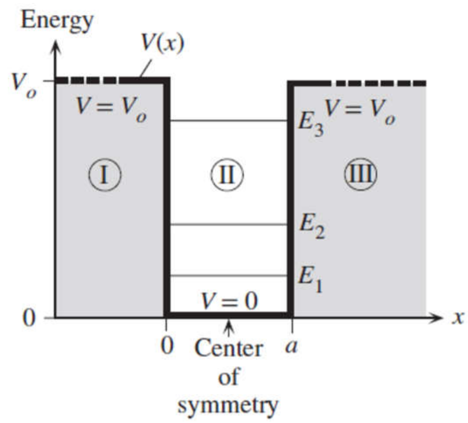
$$\frac{d^2\psi}{dx^2} + \frac{2m_e(E-V)}{\hbar^2}\psi = 0$$

- **Solution:**

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$\psi(x) = A\sin kx + B\cos kx$$

$$k = \sqrt{\frac{2m_e(E-V)}{\hbar^2}}$$




- **Note:** In the classically allowed region, we have oscillating solutions.

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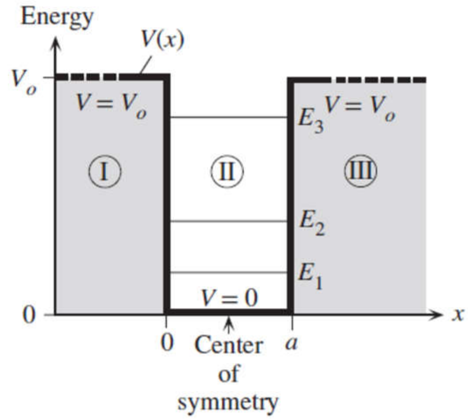
# Classically Forbidden Region

- **$E < V_0$:**
- We can divide the problem into three regions I, II, and III
- **Region II:** $V = 0$
- Schrodinger equation:

$$k^2 = \frac{2m_eE}{\hbar^2}$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$




- Solution:

$$\psi_{\text{II}}(x) = B_1\exp(jkx) + B_2\exp(-jkx)$$

- Constants $B_1$ and $B_2$ can be found from boundary conditions.

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# Classically Forbidden Region

- **Regions I and III:** $V = V_0$
- For $x \leq a$ and $x \geq a$:

$$\alpha^2 = \frac{2m_e(V_0 - E)}{\hbar^2}$$

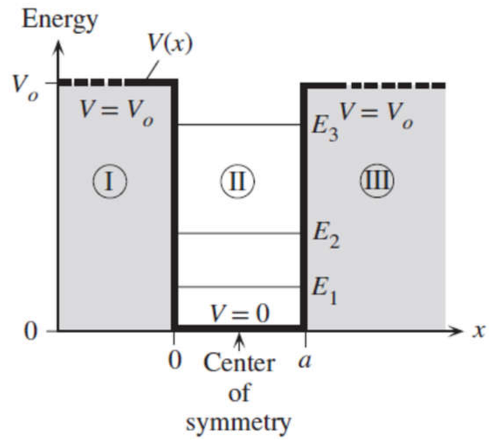
- Schrodinger equation:

$$\frac{d^2\psi}{dx^2} - \alpha^2\psi = 0$$

- Solution:

$$\psi_{\text{I}} = A_1 \exp(\alpha x) + A_2 \exp(-\alpha x)$$
$$\psi_{\text{III}} = C_1 \exp(\alpha x) + C_2 \exp(-\alpha x)$$

- $x \leq a$ : $A_2 = 0$ and $x \geq a$: $C_1 = 0$

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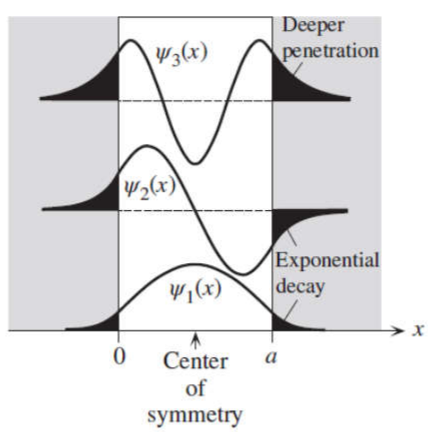
# Bound Solutions

$$\psi(x) = \begin{cases} A_1 e^{\alpha x} & \text{for } x \leq 0 \\ B_1 \cos kx + B_2 \sin kx & \text{for } 0 \leq x \leq a \\ C_2 e^{-\alpha x} & \text{for } x \geq a \end{cases}$$

$$\alpha = \sqrt{\frac{2m_e(V_0 - E)}{\hbar^2}}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

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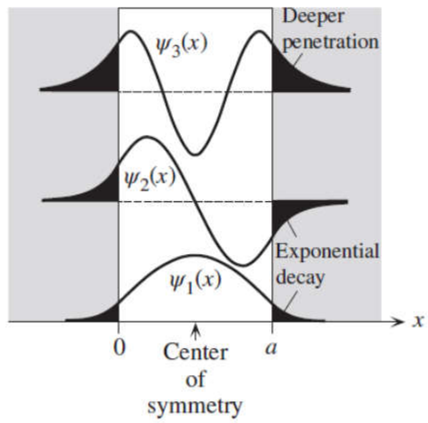
## Symmetric Wavefunctions

$$\psi(x) = \begin{cases} A_1 e^{\alpha x} & \text{for } x \le 0 \\ B_1 \cos kx & \text{for } 0 \le x \le a \\ C_2 e^{-\alpha x} & \text{for } x \ge a \end{cases}$$

$$\alpha = \sqrt{\frac{2m_e(V_0 - E)}{\hbar^2}}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$








## Symmetric Wavefunction

**At $x = a$:**

$$\psi(a) = B_1 \cos(ka) = C_2 e^{-\alpha a}$$

$$\psi'(a) = -kB_1 \sin(ka) = -C_2 \alpha e^{-\alpha a}$$

Dividing the second equation by the first gives

$$\tan(ka) = \alpha / k$$

$$\tan\left(\frac{\pi}{2}\sqrt{\frac{E}{E_L}}\right) = \sqrt{\frac{V_0 - E}{E}}$$

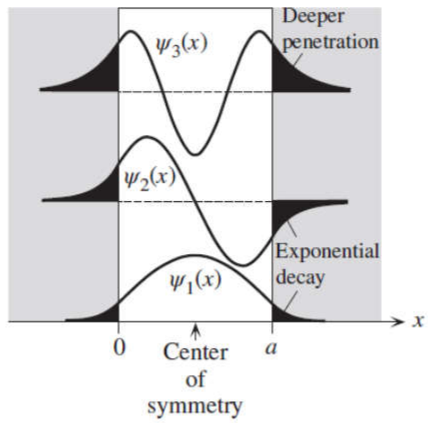
$E_L$ : infinite well ground state energy

# Anti-symmetric Wavefunctions

$$\psi(x) = \begin{cases} A_1 e^{\alpha x} & \text{for } x \le 0 \\ B_2 \sin kx & \text{for } 0 \le x \le a \\ C_2 e^{-\alpha x} & \text{for } x \ge a \end{cases}$$

$$\alpha = \sqrt{\frac{2m_e\left(V_0 - E\right)}{\hbar^2}}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$








# Antisymmetric Wavefunction

**At $x = a$:**

$$\psi(a) = B_2 \sin(ka) = C_2 e^{-\alpha a}$$

$$\psi'(a) = kB_2 \cos(ka) = -C_2 a \alpha e^{-\alpha a}$$

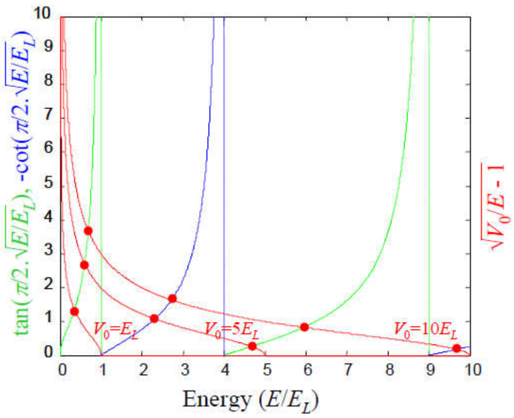
Dividing the second equation by the first gives

$$\cot\left(ka\right) = -\alpha / k$$

$$-\cot\left(\frac{\pi}{2}\sqrt{\frac{E}{E_L}}\right) = \sqrt{\frac{V_0 - E}{E}}$$

$E_L$ : infinite well ground state energy

# *Solutions*

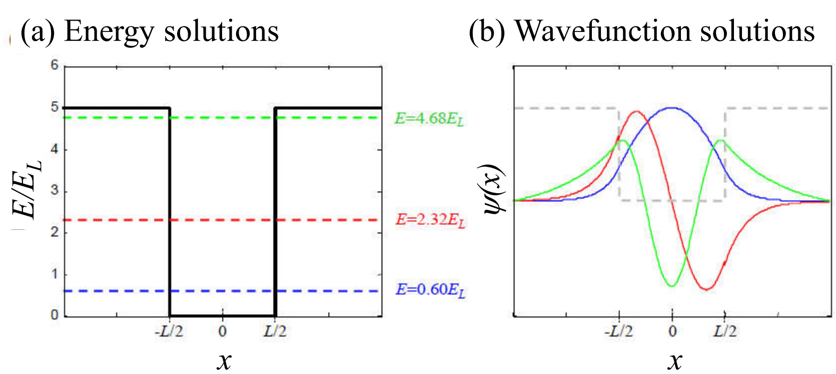



**There is always at least one bound solution no matter how shallow is the well!**





# *Solutions*




Note that The higher the energy, the lower the effective confining potential, and the greater the penetration into the barriers.