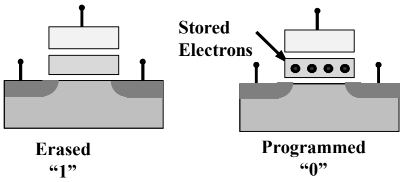
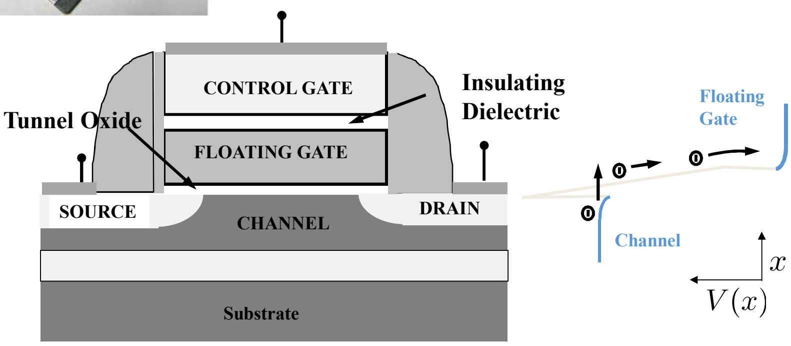
TUNNELING THROUGH A BARRIER

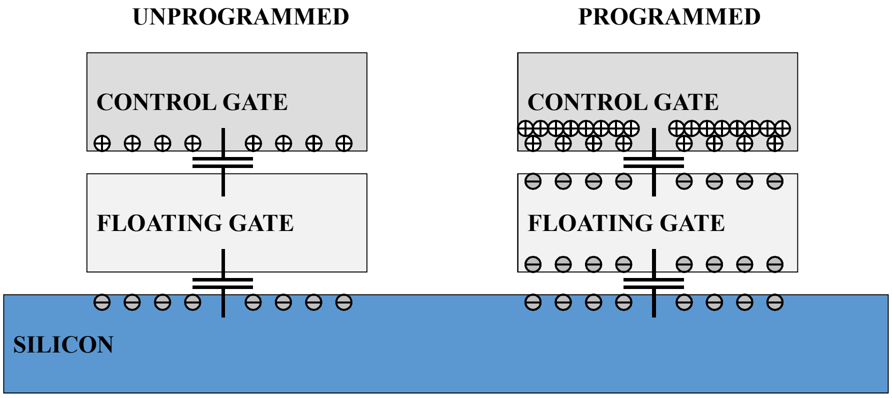
Flash Memory





Electrons tunnel preferentially when a voltage is applied

Reading Flash Memory



To obtain the same channel charge, the programmed gate needs a higher control-gate voltage than the unprogrammed gate

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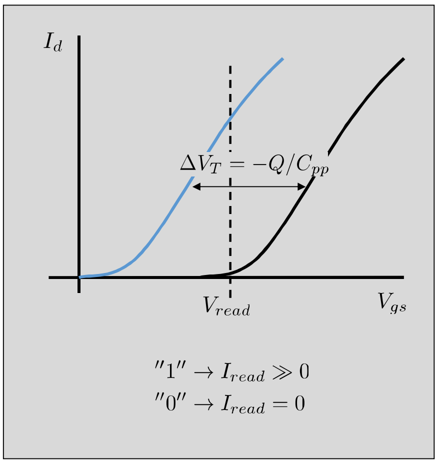
Reading Flash Memory



Reading a *bit* means:

1. Apply V_{read} on the control gate
2. Measure drain current I_d of the floating-gate transistors

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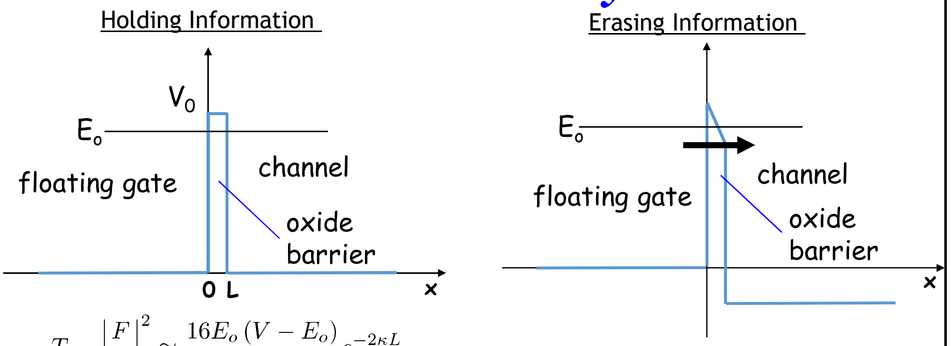
Flash Memory



$$T = \left|\frac{F}{A}\right|^2 \approx \frac{16E_o(V-E_o)}{V^2}e^{-2\kappa L}$$

Retention = the ability to hold on to the charge

Tunnel oxide thickness	Time for 20% charge loss
4.5 nm	4.4 minutes
5 nm	1 day
6 nm	½ - 6 years

Effective thickness
of the tunneling barrier
decreases, as the applied voltage
bends the potential energy levels

7-8 nm oxide thickness is the bare minimum, so that the flash memory chip can retain charge in the floating gates for at least 20 years

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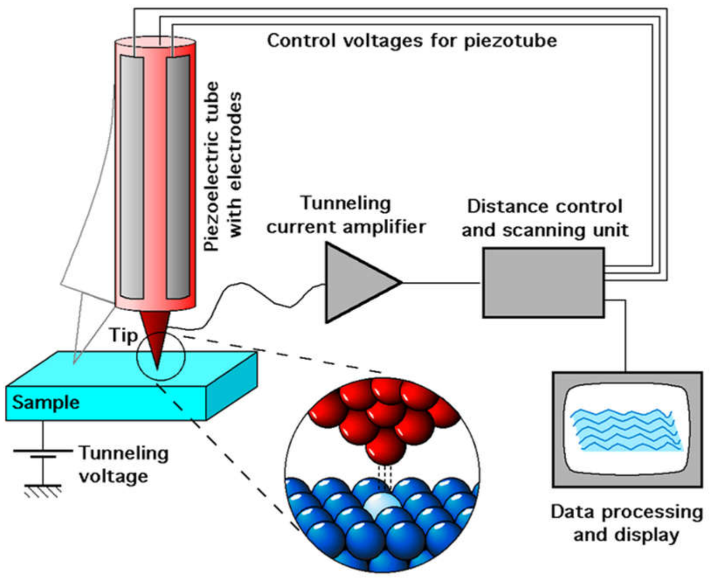
Scanning Tunneling Microscope



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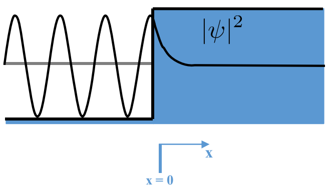
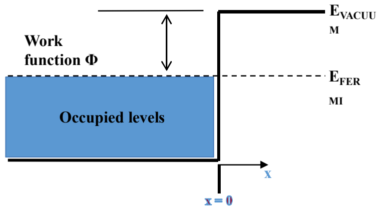
Leaky Particles

Due to "barrier penetration," the electron density of a metal actually extends outside the surface of the metal!





Assume that the work function (i.e., the energy difference between the most energetic conduction electrons and the potential barrier at the surface) of a certain metal is $\Phi = 5$ eV. Estimate the distance x outside the surface of the metal at which the electron probability density drops to 1/1000 of that just inside the metal.

$$\frac{|\psi(x)|^2}{|\psi(0)|^2} = e^{-2\kappa x} \approx \frac{1}{1000} \quad \Rightarrow \quad x = -\frac{1}{2\kappa}\ln\left(\frac{1}{1000}\right) \approx 0.3 \text{ nm}$$

using $\kappa = \sqrt{\frac{2m_e}{\hbar^2}(V_o - E)} = 2\pi\sqrt{\frac{2m_e}{h^2}\Phi} = 2\pi\sqrt{\frac{5 \text{ eV}}{1.505 \text{ eV} \cdot \text{nm}^2}} = 11.5 \text{ nm}^{-1}$

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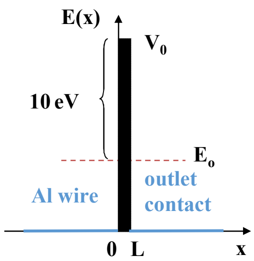
Al Wire Contacts

"Everyday" problem:

You're putting the electrical wiring in your new house, and you're considering using Aluminum wiring, which is cheap and a good conductor. However, you also know that aluminum tends to form an oxide surface layer (Al_2O_3) which can be as much as several nanometers thick.

This oxide layer could cause a problem in making electrical contacts with outlets, for example, since it presents a barrier of roughly <u>10 eV</u> to the flow of electrons in and out of the Al wire.



Your requirement is that your transmission coefficient across any contact must be $T > 10^{-10}$, or else the resistance will be too high for the high currents you're using, causing a fire risk.

<u>Should you use aluminum wiring or not?</u>

<u>Compute *L*:</u>

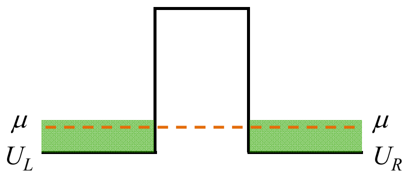
$$T \approx e^{-2\kappa L} \approx 10^{-10} \quad \Rightarrow \quad L \approx -\frac{1}{2\kappa}\ln(10^{-10}) \approx 0.72 \text{ nm}$$

$$\kappa = \sqrt{\frac{2m_e}{\hbar^2}(V_o - E)} = 2\pi\sqrt{\frac{2m_e}{h^2}(V_o - E)} = 2\pi\sqrt{\frac{10 \text{ eV}}{1.505 \text{ eV} \cdot \text{nm}^2}} = 16 \text{ nm}^{-1}$$

Oxide is thicker than this, so go with Cu wiring! (Al wiring in houses is illegal for this reason)

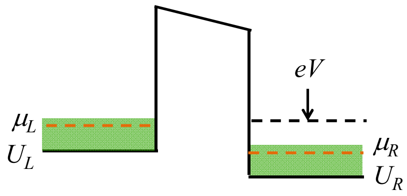
Current-Voltage Relation

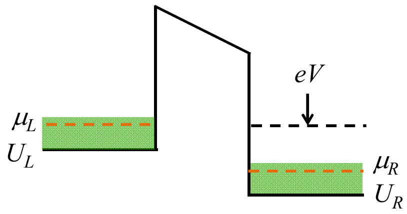
- Consider a barrier surrounded by a Fermi sea of electrons.



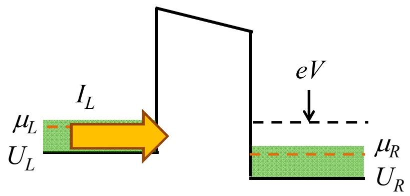
- **Equilibrium:** The distribution of electrons is given by a Fermi function.

- **Non-equilibrium:**





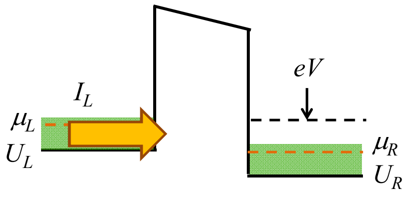
Current from Left



$$I_L = 2e\int_0^{\infty} f\left[\varepsilon(k), \mu_L\right] v(k) T(k) \frac{dk}{2\pi}$$

- **2** : two spins
- ***e*** : converts number current into electrical current
- ***f*** **[*ε*(*k*), *μ*$_L$]** : probability that each state is occupied
- ***v*(*k*)** : turns the charge density into a current density
- ***T*(*k*)** : probability that an incident electron passes through
- ***dk*** **/ 2*π*** : counting *k*-states

Current from Left



- Integration over energy:

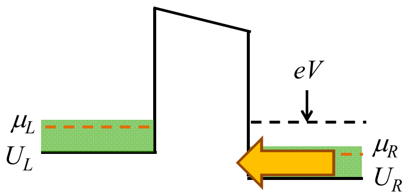
$$dk = \frac{dk}{dE} dE$$

$$k = \frac{\omega}{v} = \frac{E}{\hbar v} \Rightarrow dk = \frac{dE}{\hbar v}$$

$$I_L = 2e\int_{U_L}^{\infty} f\left(E, \mu_L\right) v T(E) \frac{dE}{2\pi\hbar v}$$

$$= (2e/h)\int_{U_L}^{\infty} f\left(E, \mu_L\right) T(E)\, dE$$

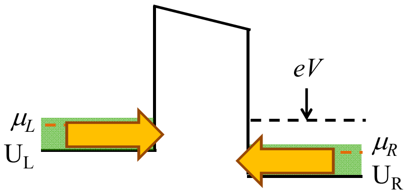
Current from Right



$$I_R = -\frac{2e}{h}\int_{U_R}^{\infty} f\left(E, \mu_R\right) T(E)\, dE$$

- The transmission coefficient is the same from both sides of a barrier, so the same $T(E)$ appears in I_L and I_R.

Total Current

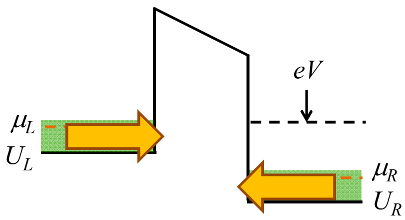


$$I = I_L + I_R$$

$$= \frac{2e}{h}\int_{U_L}^{\infty}\left[f\left(E,\mu_L\right)-f\left(E,\mu_R\right)\right]T(E)\,dE$$

- **The current is not simply proportional to the bias.**
- $I = 0$ when $V = 0$.

Limits: Large Bias

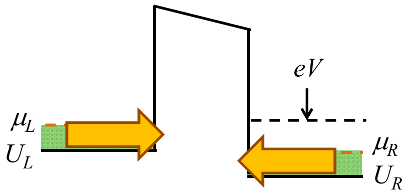


$$I \approx I_L$$

$$= \frac{2e}{h}\int_{U_L}^{\infty} f\left(E,\mu_L\right)T(E)\,dE$$

- **Right-hand electrons play no role at all.**

Limits: Low Temperature

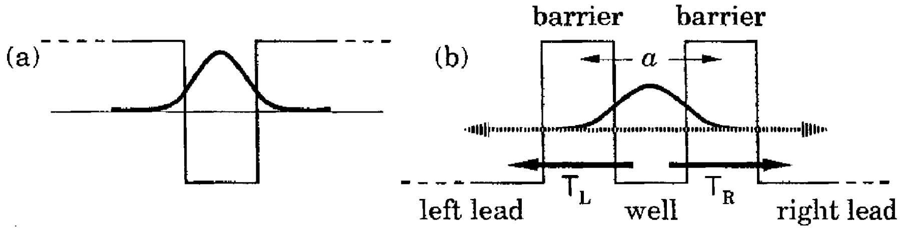


$$I = I_L + I_R$$
$$= \frac{2e}{h}\int_{\mu_R}^{\mu_L} T(E)\,dE$$

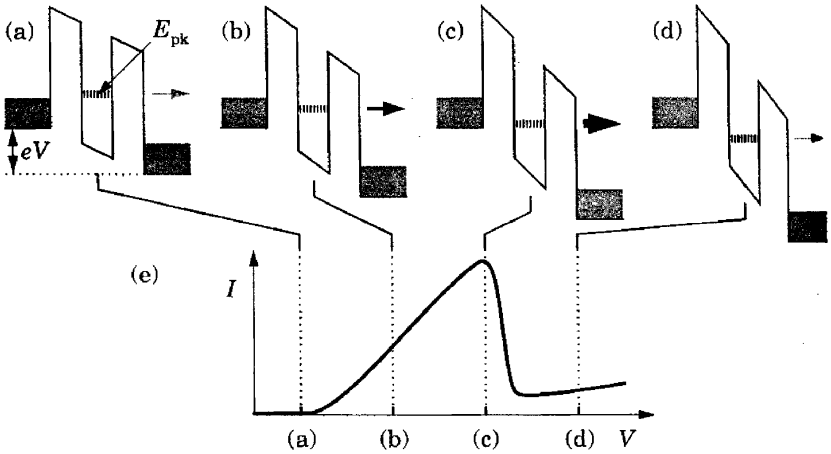
- Fermi distribution can be approximated by step functions.

Resonant Tunneling

- In a well between two barriers, there are no longer true bound states.
- The electron may remain in the well for a long time if the barriers are thick enough.
- T of double barriers rises dramatically near resonance → **resonant tunneling**.



Resonant Tunneling Diode



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Transmission Coefficient

- We can solve the transmission using the *T*-matrix for the structure.

$$t = \frac{t_L t_R}{1 - r_L r_R \exp(2ika)} \qquad r_L = |r_L| e^{i\rho_L}$$

$$T = |t|^2 = \frac{T_L T_R}{1 + R_L R_R - 2\sqrt{R_L R_R}\cos(2ka + \rho_L + \rho_R)}$$

$$= \frac{T_L T_R}{\left(1 - \sqrt{R_L R_R}\right)^2 + 4\sqrt{R_L R_R}\sin^2\frac{\phi}{2}} \qquad \phi = 2ka + \rho_L + \rho_R$$

- **Condition for resonance:**

$$\phi = 2ka + \rho_L + \rho_R = 2n\pi \rightarrow \textit{Constructive interference}$$

$$T = T_{\text{peak}} = \frac{T_L T_R}{\left(1 - \sqrt{R_L R_R}\right)^2}$$

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Partial Waves

- We will add the partial waves → used in Fabry-Perot cavity lasers.

$$t = t'_L e^{ika} t_R + t'_L e^{ika} r_R e^{ika} r_L e^{ika} t_R + t'_L e^{ika} r_R e^{ika} r_L e^{ika} r_R e^{ika} r_L e^{ika} t_R + \cdots$$

$$t = t'_L e^{ika} t_R \left[1 + r_L r_R e^{2ika} + \left(r_L r_R e^{2ika} \right)^2 + \cdots \right] = \frac{t_L t_R e^{ika}}{1 - r_L r_R e^{2ika}}$$