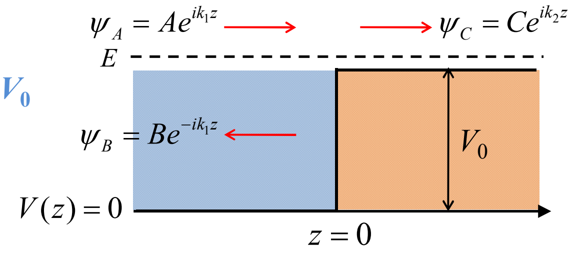
TUNNELING TRANSPORT

Schrodinger Equations

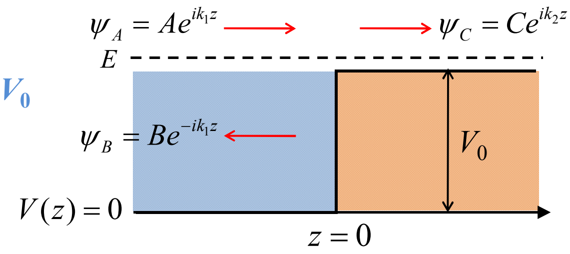
- **Case I: $E > V_0$**



- In region 1: $E\psi = \dfrac{\hbar^2}{2m}\dfrac{\partial^2 \psi}{\partial z^2}$ ⇒ $k_1^2 = \dfrac{2mE}{\hbar^2}$
- In region 2: $(E - V)\psi = \dfrac{\hbar^2}{2m}\dfrac{\partial^2 \psi}{\partial z^2}$ ⇒ $k_2^2 = \dfrac{2m(E - V_0)}{\hbar^2}$

Boundary Conditions

- **Case I: $E > V_0$**



$$\psi_1 = Ae^{ik_1 z} + Be^{-ik_1 z} \qquad \psi_2 = Ce^{ik_2 z}$$

- ψ is continuous: $\psi_1(0) = \psi_2(0)$ ⇒ $A + B = C$
- $\frac{\partial \psi}{\partial z}$ is continuous: $\frac{\partial}{\partial z}\psi_1(0) = \frac{\partial}{\partial z}\psi_2(0)$ ⇒ $A - B = \frac{k_2}{k_1} C$

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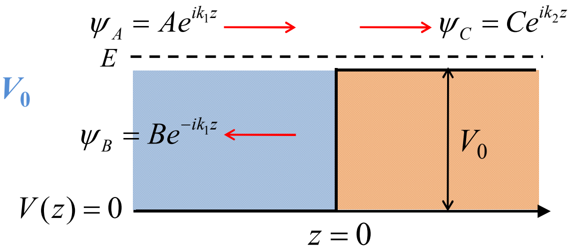
Ratios

- **Case I: $E > V_0$**



$$\left.\begin{aligned} A + B &= C \\ A - B &= \frac{k_2}{k_1} C \end{aligned}\right\} \Rightarrow \quad \begin{aligned} \frac{B}{A} &= \frac{1 - k_2/k_1}{1 + k_2/k_1} = \frac{k_1 - k_2}{k_1 + k_2} \\ \frac{C}{A} &= \frac{2}{1 + k_2/k_1} = \frac{2k_1}{k_1 + k_2} \end{aligned}$$

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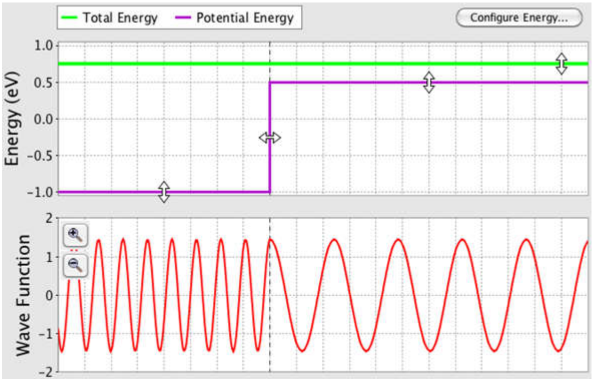
Wavefunctions



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Quantum Electron Currents

Given an electron of mass m

that is located in space with charge density $\rho = q\left|\psi(x)\right|^2$

and moving with momentum $\langle p \rangle$ corresponding to $\langle v \rangle = \hbar k / m$

… then the current density for a *single electron* is given by

$$J = \rho v = q\left|\psi\right|^2 (\hbar k / m)$$

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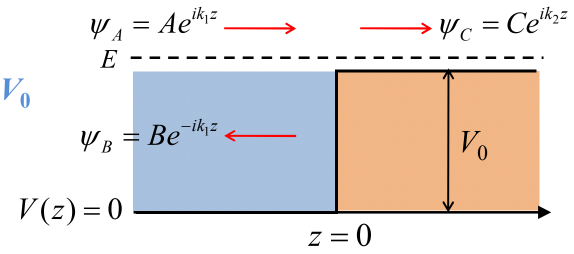
Reflection and Transmission

- **Case I: $E > V_0$**



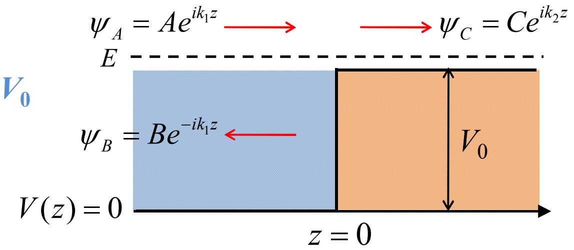
$$\text{Reflection} = R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \frac{J_B}{J_A} = \frac{|\psi_B|^2 (\hbar k_1 / m)}{|\psi_A|^2 (\hbar k_1 / m)} = \left|\frac{B}{A}\right|^2$$

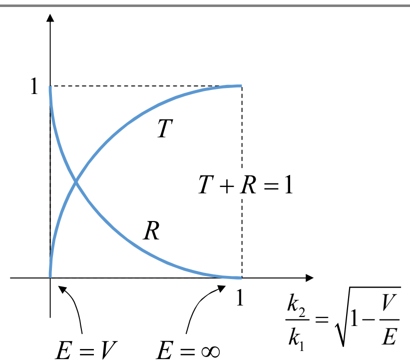
$$\text{Transmission} = T = \frac{J_{\text{transmitted}}}{J_{\text{incident}}} = \frac{J_C}{J_A} = \frac{|\psi_C|^2 (\hbar k_2 / m)}{|\psi_A|^2 (\hbar k_1 / m)} = \left|\frac{C}{A}\right|^2 \frac{k_2}{k_1}$$

$$\frac{B}{A} = \frac{k_1 - k_2}{k_1 + k_2} \qquad \frac{C}{A} = \frac{2k_1}{k_1 + k_2}$$

Reflection and Transmission

- **Case I: $E > V_0$**



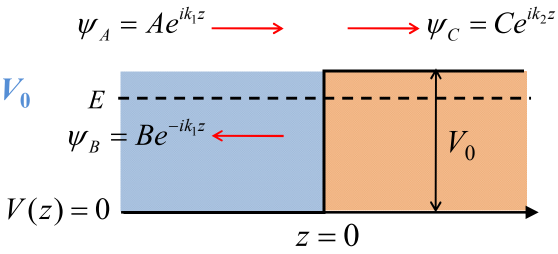


$$\text{Reflection} = R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

$$\text{Transmission} = T = 1 - R = \frac{4k_1 k_2}{|k_1 + k_2|^2}$$

Tunneling Transport

- **Case II: $E < V_0$**



- In region 1: $E\psi = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial z^2}$ ➡ $k_1^2 = \frac{2mE}{\hbar^2}$
- In region 2: $(E - V)\psi = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial z^2}$ ➡ $k_2^2 = \frac{2m(E - V_0)}{\hbar^2}$

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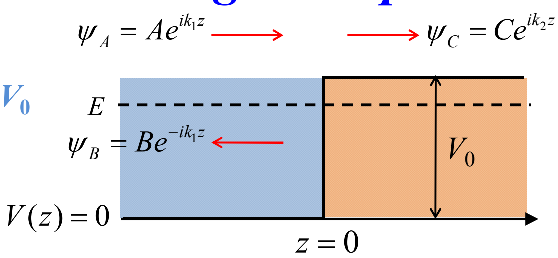
Tunneling Transport

- **Case II: $E < V_0$**



- In region 1: $E\psi = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial z^2}$ ➡ $k_1^2 = \frac{2mE}{\hbar^2}$
- In region 2: $(E - V)\psi = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial z^2}$ ➡ $\kappa_2^2 = \frac{2m(V_0 - E)}{\hbar^2}$

$k_2 = i\kappa_2$

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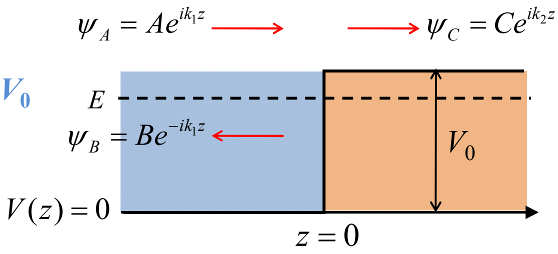
Tunneling Transport

- **Case II: $E < V_0$**

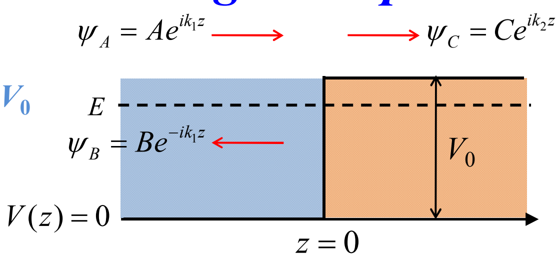


$$\psi_1 = Ae^{ik_1 z} + Be^{-ik_1 z} \qquad \psi_2 = Ce^{-\kappa_2 z}$$

- ψ is continuous: $\psi_1(0) = \psi_2(0)$ ⇒ $A + B = C$
- $\dfrac{\partial \psi}{\partial z}$ is continuous: $\dfrac{\partial}{\partial z}\psi_1(0) = \dfrac{\partial}{\partial z}\psi_2(0)$ ⇒ $A - B = i\dfrac{\kappa_2}{k_1}C$

Tunneling Transport

- **Case II: $E < V_0$**



$$\psi_1 = Ae^{ik_1 z} + Be^{-ik_1 z} \qquad \psi_2 = Ce^{-\kappa_2 z}$$

$$\left.\begin{aligned} &A + B = C \\ &A - B = i\frac{\kappa_2}{k_1}C \end{aligned}\right\} \Rightarrow \quad \frac{B}{A} = \frac{1 - i\kappa_2/k_1}{1 + i\kappa_2/k_1}, \qquad \frac{C}{A} = \frac{2}{1 + i\kappa_2/k_1}$$

$$R = \left|\frac{B}{A}\right|^2 = 1$$

$$T = 0$$

Total reflection → Transmission must be zero

Reflection of EM Waves and QM Waves

$$P = \hbar\omega \times \frac{\text{photons}}{\text{s} \times \text{cm}^2}$$

$$P = \frac{|E|^2}{\eta}$$

$$R = \frac{P_{\text{reflected}}}{P_{\text{incident}}} = \left|\frac{E_0^r}{E_0^i}\right|^2$$

Then for optical material when $\mu = \mu_0$:

$$R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

$$= \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2$$

= probability of a particular photon being reflected

$$J = q \times \frac{\text{electrons}}{\text{s} \times \text{cm}^2}$$

$$J = \rho v = q|\psi|^2 (\hbar k / m)$$

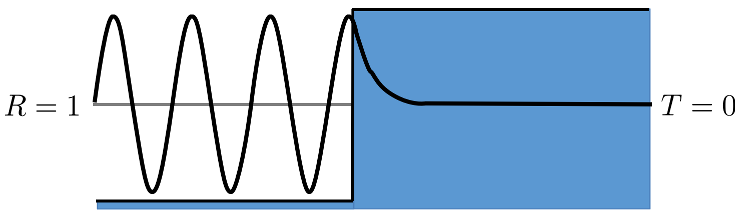
$$R = \frac{J_{\text{reflected}}}{J_{\text{incident}}} = \frac{|\psi_B|^2}{|\psi_A|^2}$$

$$R = \left|\frac{B}{A}\right|^2 = \left|\frac{k_1 - k_2}{k_1 + k_2}\right|^2$$

= probability of a particular electron being reflected

Tunneling Through Potential Barriers

Total Reflection at Boundary



Frustrated Total Reflection (Tunneling)