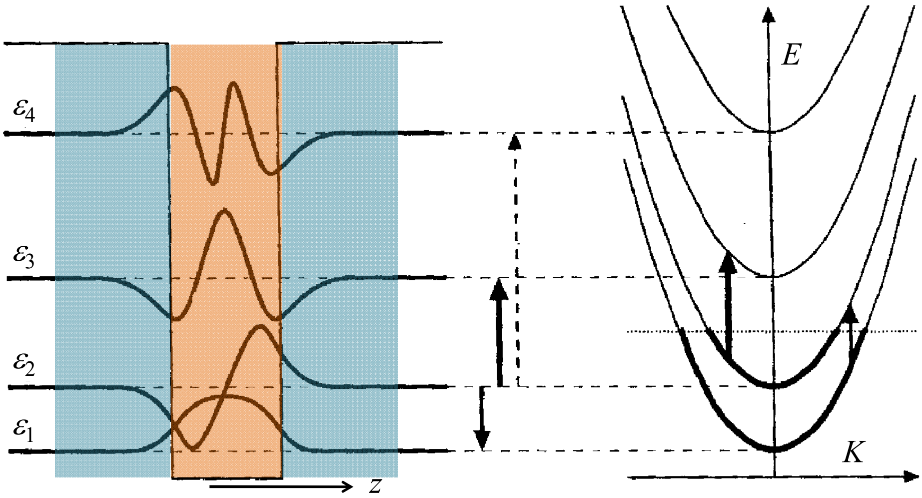
ABSORPTION IN A QUANTUM WELL

Quantum Well



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Quantum Well

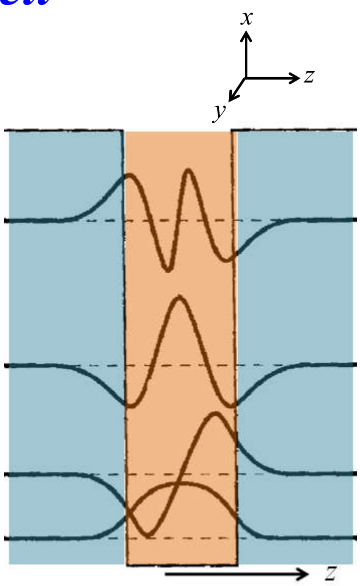
- States factorize into a product of a bound state in z and a transverse plane wave.

$$\psi_{i\vec{K}}\left(\vec{R}\right) = A^{-1/2}\phi_i(z)\,e^{i\vec{K}\cdot\vec{r}}$$

$$\vec{r} = (x, y)$$

$$E_i\left(\vec{K}\right) = \varepsilon_i + \frac{\hbar^2 k^2}{2m}$$

- Practically, we need to use the effective-mass wavefunctions.



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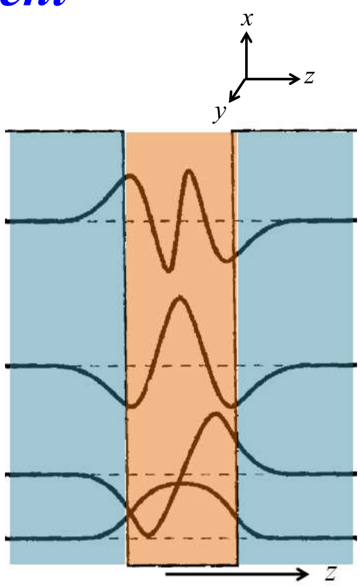
Matrix Element

- Matrix element between states i and j:

$$\left\langle j\vec{K}' \,|\, \vec{e}\cdot\hat{p} \,|\, i\vec{K} \right\rangle$$

- **This depends strongly on the polarization of light.**



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Polarization of Light

- Let $\vec{e} = (1,0,0)$

$$\vec{e}\cdot\hat{p} = -i\hbar\frac{\partial}{\partial x}$$

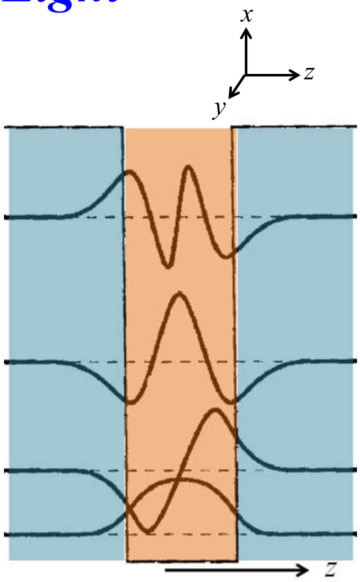
$$\left(\vec{e}\cdot\hat{p}\right)\psi_{i\vec{K}}\left(\vec{R}\right) = -i\hbar\frac{\partial}{\partial x}\left[A^{-1/2}\phi_i(z)e^{i\vec{K}\cdot\vec{r}}\right]$$

$$= \hbar k_x \psi_{i\vec{K}}\left(\vec{R}\right)$$

- Matrix element:

$$\left\langle j\vec{K}'\,|\,\vec{e}\cdot\hat{p}\,|\,i\vec{K}\right\rangle = \hbar k_x\left\langle j\vec{K}'\,|\,i\vec{K}\right\rangle = 0$$

No light is absorbed with this polarization!



Polarization of Light

- Let $\vec{e} = (0,0,1)$

$$\vec{e}\cdot\hat{p} = -i\hbar\frac{\partial}{\partial z}$$

$$\left(\vec{e}\cdot\hat{p}\right)\psi_{i\vec{K}}\left(\vec{R}\right) = -i\hbar\frac{\partial}{\partial z}\left[A^{-1/2}\phi_i(z)e^{i\vec{K}\cdot\vec{r}}\right]$$

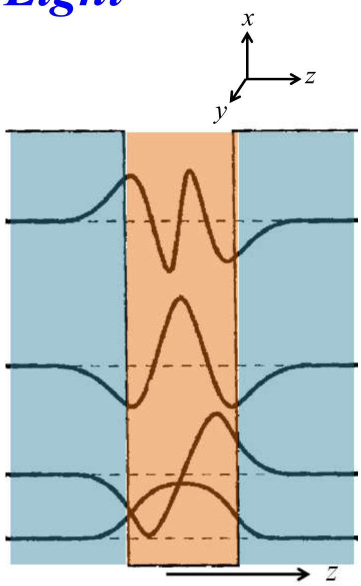
$$= A^{-1/2}e^{i\vec{K}\cdot\vec{r}}\,\hat{p}_z\phi_i(z)$$

$$= -i\hbar\,A^{-1/2}e^{i\vec{K}\cdot\vec{r}}\,\frac{\partial}{\partial z}\phi_i(z)$$

$$\left\langle j\vec{K}'\,|\,\vec{e}\cdot\hat{p}\,|\,i\vec{K}\right\rangle$$

$$= \frac{1}{A}\int dz\int d^2r\,\phi_j^*(z)e^{i(K-K')\cdot r}\,\hat{p}_z\phi_i(z)$$

Light is absorbed with this polarization!

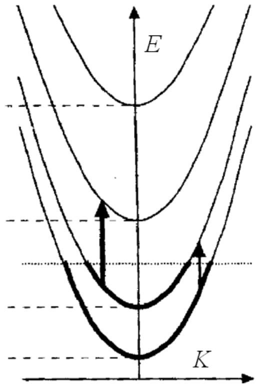


Conductivity

$$\sigma_1(\omega) = \frac{\pi e^2}{m_0^2 \omega} \frac{2}{\Omega} \sum_{i,j} \left| \langle j | \vec{e} \cdot \hat{p} | i \rangle \right|^2$$
$$\times \left\{ f\left[E_i\left(\vec{K}\right)\right] - f\left[E_j\left(\vec{K}\right)\right] \right\} \delta\left(E_j\left(\vec{K}\right) - E_i\left(\vec{K}\right) - \hbar\omega\right)$$

- Simplifications
  1) If subbands have equal mass:

$$E_j(\vec{K}) - E_i(\vec{K}) = \varepsilon_j - \varepsilon_i$$
$$\varepsilon_j - \varepsilon_i = \hbar\omega_{ji}$$
$$\delta\left[E_j(\vec{K}) - E_i(\vec{K}) - \hbar\omega\right] = \hbar\delta\left[\omega_{ji} - \omega\right]$$



Simplifications

2) Sums over the occupation functions count the density of electrons:

$$\frac{2}{A} \sum_{\vec{K}} f\left[E_j\left(\vec{K}\right)\right] = n_j$$

3) The ω in the prefactor can be replaced by ω_{ji}:

$$\omega \rightarrow \omega_{ji}$$

4) Introduce oscillator strengths:

$$f_{ji} = \frac{2}{m\hbar\omega_{ji}} \left| \langle j | \hat{p}_z | i \rangle \right|^2$$

Conductivity

$$\sigma_1(\omega) = \frac{\pi e^2}{2mL} \sum_{\substack{i,j \\ j \neq i}} f_{ji} \left(n_i - n_j \right) \delta\left(\omega_{ji} - \omega \right)$$

- We omit transitions from a state to itself.
- Absorption is in the frequencies corresponding to the separation of bound states in the well.

  → Discrete absorption lines
- **Note:** Lines will be broadened by any difference in effective mass between subbands.

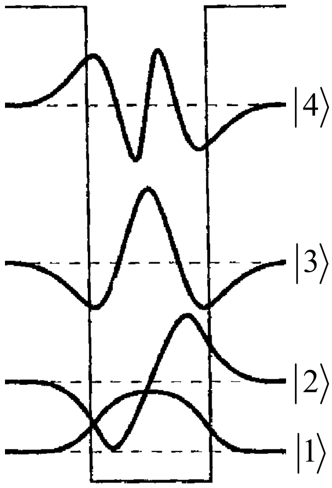
Strength of Transition

$$\sigma_1(\omega) = \frac{\pi e^2}{2mL} \sum_{\substack{i,j \\ j \neq i}} f_{ji} \left(n_i - n_j \right) \delta\left(\omega_{ji} - \omega \right)$$

- Transition strength can be manipulated
  - Through f_{ji} by changing the shape of the quantum well.
  - By modifying the occupations through doping, injection of carriers, pumping, or simply a change in temperature.
- A population inversion ($n_j > n_i$) can make $\sigma_i(\omega) < 0$ in some range of frequencies.

Selection Rule

- Symmetric well: $V(-z) = V(z)$
- The wavefunctions in a symmetric well are either even or odd.
- The derivative changes the parity and the matrix element will be non-zero only if one state is even and the other odd.
- The absorption is permitted from $n = 1$ to $n = 2, 4, \ldots$ but not to odd values of n.



Selection Rule

- Consider an infinite deep quantum well.

$$|1\rangle \rightarrow |2\rangle$$

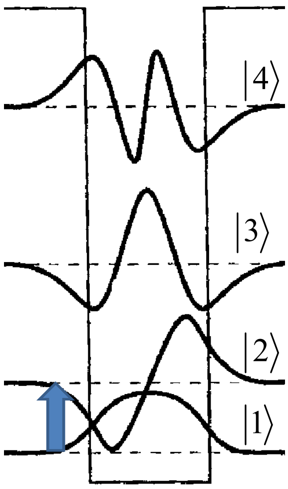
$$\langle 2|\hat{p}_z|1\rangle = -i\hbar\frac{2}{a}\int_0^a \sin\frac{2\pi z}{a}\left(\frac{d}{dz}\sin\frac{\pi z}{a}\right)dz$$

$$= -i\hbar\frac{2\pi}{a^2}\int_0^a \sin\frac{2\pi z}{a}\cos\frac{\pi z}{a}dz$$

$$= -i\hbar\frac{\pi}{a^2}\int_0^a\left(\sin\frac{3\pi z}{a} + \sin\frac{\pi z}{a}\right)dz$$

$$= -i\hbar\frac{\pi}{a^2}\left[-\frac{a}{3\pi}\cos\frac{3\pi z}{a}\bigg|_0^a - \frac{a}{\pi}\cos\frac{\pi z}{a}\bigg|_0^a\right]$$

$$= i\frac{\hbar}{a}\left(-\frac{2}{3} - 2\right) = -i\frac{8\hbar}{3a}$$



Selection Rule

$|1\rangle \rightarrow |3\rangle$

$$\langle 3|\,\hat{p}_z\,|1\rangle = -i\hbar\frac{2}{a}\int_0^a \sin\frac{3\pi z}{a}\left(\frac{d}{dz}\sin\frac{\pi z}{a}\right)dz$$

$$= -i\hbar\frac{2\pi}{a^2}\int_0^a \sin\frac{3\pi z}{a}\cos\frac{\pi z}{a}\,dz$$

$$= -i\hbar\frac{\pi}{a^2}\int_0^a\left(\sin\frac{4\pi z}{a} + \sin\frac{2\pi z}{a}\right)dz$$

$$= -i\hbar\frac{\pi}{a^2}\left[-\frac{a}{4\pi}\cos\frac{4\pi z}{a}\bigg|_0^a - \frac{a}{2\pi}\cos\frac{2\pi z}{a}\bigg|_0^a\right]$$

$$= 0$$