ELECTRON-PHOTON INTERACTION

Photon Absorption Rate

$$V(\vec{R},t) = \frac{eE_0}{im_0\omega}\left[e^{i(\vec{Q}\cdot\vec{R}-\omega t)} - e^{-i(\vec{Q}\cdot\vec{R}-\omega t)}\right](\vec{e}\cdot\hat{p})$$

- The transition rate from a state *i* to another *j* due to the absorption of a photon

$$W_{ji} = \frac{2\pi}{\hbar}\left(\frac{eE_0}{m_0\omega}\right)^2 \left|\langle j\,|\,\vec{e}\cdot\hat{p}\,|\,i\rangle\right|^2 \delta\left(E_j - E_i - \hbar\omega\right)$$

$$\vec{e}\cdot\hat{p} = -i\hbar\left(e_x\frac{\partial}{\partial x} + e_y\frac{\partial}{\partial y} + e_z\frac{\partial}{\partial z}\right)$$

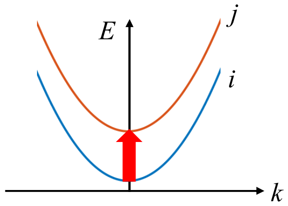
Power Absorbed

- Power absorbed in $i \rightarrow j$ transition = $W_{ji}\,\hbar\omega$
- Total Power absorbed, $P_+ = \hbar\omega \sum_{i,j} W_{ji}$
- Include the Fermi distribution:

$$P_+ = \hbar\omega \sum_{i,j} W_{ji} f(E_i)\left[1 - f(E_j)\right]$$

$f(E_i)$: Initial state is filled

$1 - f(E_j)$: Final state is empty



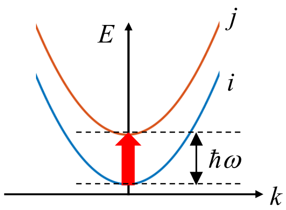
- Consider the spin degeneracy: $P_+ = 2\hbar\omega \sum_{i,j} W_{ji} f(E_i)\left[1 - f(E_j)\right]$

3

Power Absorption and Emission

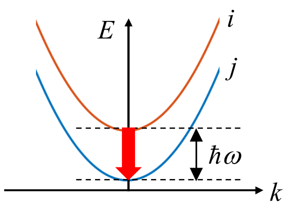
- Absorption:

$$P_+ = \frac{2\pi}{\hbar}\hbar\omega\left(\frac{eE_0}{m_0\omega}\right)^2 2\sum_{i,j}\left|\langle j \,|\, \vec{e}\cdot\hat{p} \rangle i\right|^2 \times f(E_i)\left[1 - f(E_j)\right]\delta(E_j - E_i - \hbar\omega)$$



- Emission:

$$P_- = -\frac{2\pi}{\hbar}\hbar\omega\left(\frac{eE_0}{m_0\omega}\right)^2 2\sum_{i,j}\left|\langle j \,|\, \vec{e}\cdot\hat{p} \rangle i\right|^2 \times f(E_i)\left[1 - f(E_j)\right]\delta(E_j - E_i + \hbar\omega)$$



4

Net Power Absorption

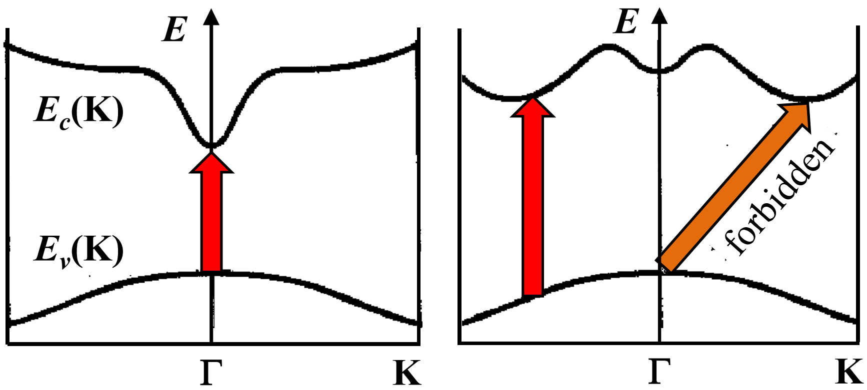
$$f(E_i)\left[1-f(E_j)\right]-f(E_j)\left[1-f(E_i)\right]=f(E_i)-f(E_j)$$

$$P=\frac{2\pi}{\hbar}\hbar\omega\left(\frac{eE_0}{m_0\omega}\right)^2 2\sum_{i,j}\left|\langle j\,|\,\vec{e}\cdot\hat{p}\rangle i\right|^2$$
$$\times\left[f(E_i)-f(E_j)\right]\delta\left(E_j-E_i-\hbar\omega\right)$$

5

Optical Absorption

- Band gaps can be measured by optical absorption.

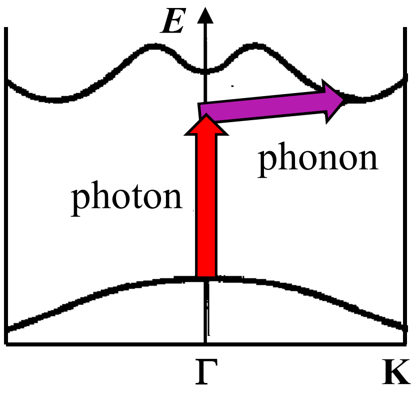


$$E_f=E_i+\hbar\omega,\qquad \vec{K}_f=\vec{K}_i+\vec{Q}$$

6

Optical Absorption

- Combination of photon and phonon transitions across the bandgap.
- Direct bandgap materials, e.g., GaAs, are preferable to indirect bandgap materials, e.g., Si, for making lasers or detectors.



7

Interband Absorption

- **Classical Approach:**
- Consider a light wave -

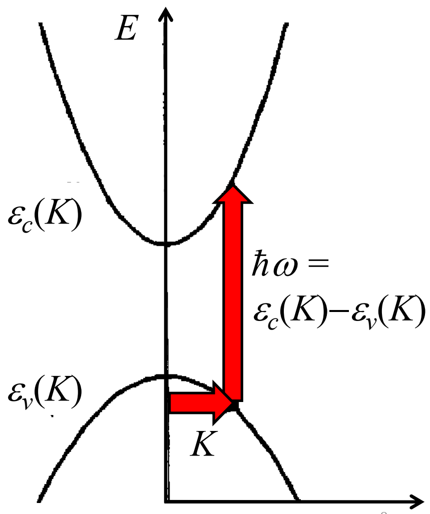
$$\vec{E}\left(\vec{R},t\right)=2\vec{e}E_0\cos\left(\vec{Q}\cdot\vec{R}-\omega t\right)$$

- $\vec{E}$ induces a current with in-phase component -

$$\vec{J}=\sigma_1\vec{E}$$

- Energy dissipation rate -

$$\vec{J}\cdot\vec{E}$$



8

Classical Approach

- **Classical Approach:**
- Consider a light wave -

$$\vec{E}\left(\vec{R},t\right)=2\vec{e}E_0\cos\left(\vec{Q}\cdot\vec{R}-\omega t\right)$$

- $\vec{E}$ induces a current with in-phase component -

$$\vec{J}=\sigma_1\vec{E}$$

- Energy dissipation rate -

$$\vec{J}\cdot\vec{E}$$

- **Total power dissipated:**

$$P=\Omega\frac{1}{t}\int_0^t\vec{J}\cdot\vec{E}\,dt'$$
$$=\frac{4\sigma_1\Omega E_0^2}{t}\int_0^t\cos^2\left(\vec{Q}\cdot\vec{R}-\omega t'\right)dt'$$
$$=\frac{2\sigma_1\Omega E_0^2}{t}\int_0^t\left[1+\cos 2\left(\vec{Q}\cdot\vec{R}-\omega t'\right)\right]dt'$$
$$=2\sigma_1\Omega E_0^2$$

9

Conductivity

- Classical approach:

$$P=2\sigma_1\Omega E_0^2$$

- Quantum mechanical approach:

$$P=\frac{2\pi}{\hbar}\hbar\omega\left(\frac{eE_0}{m_0\omega}\right)^2 2\sum_{i,j}\left|\left\langle j\,|\,\vec{e}\cdot\hat{p}\right\rangle i\right|^2$$
$$\times\left[f\left(E_i\right)-f\left(E_j\right)\right]\delta\left(E_j-E_i-\hbar\omega\right)$$

$$\sigma_1(\omega)=\frac{\pi e^2}{m_0^2\omega}\frac{2}{\Omega}\sum_{i,j}\left|\left\langle j\,|\,\vec{e}\cdot\hat{p}\,|\,i\right\rangle\right|^2\left[f\left(E_i\right)-f\left(E_j\right)\right]\delta\left(E_j-E_i-\hbar\omega\right)$$

10