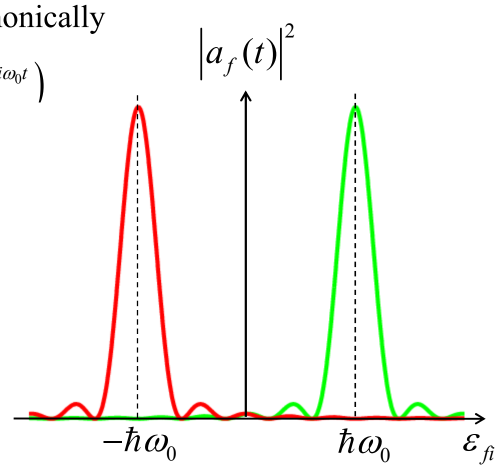
PHONON SCATTERING

Golden Rule for Oscillating Potential

- If the perturbation varies harmonically

$$V(t) = 2\tilde{V}\cos\omega_0 t = \tilde{V}\left(e^{-i\omega_0 t} + e^{i\omega_0 t}\right)$$



Absorption:

$$W_{fi} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \delta\left(\varepsilon_f - \varepsilon_i - \hbar\omega_0\right)$$

Emission:

$$W_{fi} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \delta\left(\varepsilon_f - \varepsilon_i + \hbar\omega_0\right)$$

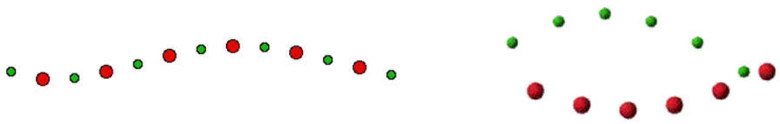
2

Fermi's Golden Rule

- Two important applications:
  - Scattering by phonons (lattice vibration)
  - Scattering by photons (light)
- Note that
  - Phonons and photons are quantized.
  - We take the perturbation classically.
  - Scattering due to a single phonon is multiplied by the Bose-Einstein distribution.

Phonon Scattering

- **Deformation coupling**
  → Longitudinal acoustic phonon
- **Polar coupling**
  → Longitudinal optical phonon



Longitudinal Acoustic Phonons

- Compress and dilate alternating regions of the solid
  → Deformation potential
- Electron-phonon coupling is through deformation potential.
- The constant of proportionality of the change in the potential to the strain is called deformation potential *C*.
- Strain: $e(z) = \dfrac{u_j - u_{j-1}}{a} \rightarrow \dfrac{\partial u}{\partial z}$

Deformation Potential

- Displacement of an atom: $u(z) = U_0 \cos\left(qz - \omega_q t\right)$
- Longitudinal strain: $e(z) = \dfrac{\partial u}{\partial z} = -U_0 q \sin\left(qz - \omega_q t\right)$
- Perturbing potential: $\hat{V}(z,t) = Ce(z) = -U_0 qC \sin\left(qz - \omega_q t\right)$
  $= iU_0 qC\left(e^{iqz}e^{-i\omega_q t} - e^{-iqz}e^{i\omega_q t}\right)$
- In three dimensions: $\hat{V}(\vec{R},t) = iU_0 qC\left(e^{i\vec{Q}\cdot\vec{R}}e^{-i\omega_{\vec{Q}} t} - e^{-i\vec{Q}\cdot\vec{R}}e^{i\omega_{\vec{Q}} t}\right)$

Number of Phonons

- Bose-Einstein distribution:

$$N_{\vec{Q}} = \frac{1}{e^{\frac{\hbar\omega_Q}{k_B T}} - 1}$$

$\rightarrow$ Number of phonons in a mode with wave-vector Q.

- **Phonon absorption:** Rate multiplied by N_Q.
- **Phonon emission:** Rate multiplied by $N_Q + 1$.
  - N_Q is due to stimulated emission.
  - Extra phonon is due to spontaneous emission.

7

Matrix Element

- Consider electrons in a finite box of volume Ω.
- Wavefunctions are plane waves, scattering from K to K′.

$$\psi_i = \Omega^{-1/2} e^{i\vec{K}\cdot\vec{R}}, \quad \psi_f = \Omega^{-1/2} e^{i\vec{K}'\cdot\vec{R}}$$

- First, we will consider only the first term of the perturbation:

$$\hat{V}(\vec{R},t) = iU_0 qC\left(e^{i\vec{Q}\cdot\vec{R}} e^{-i\omega_{\vec{Q}} t}\right)$$

$\exp\left(-i\omega_Q t\right)$:

- Causes the electron to absorb energy from phonon.
- Energy conservation $\rightarrow$ $\varepsilon\left(\vec{K}'\right) = \varepsilon\left(\vec{K}\right) + \hbar\omega_{\vec{Q}}$

8

Matrix Element

- The matrix element

$$V^{+}_{\vec{K}',\vec{K}} = \int_{\Omega} \psi^{*} \hat{V} \psi \, d^{3}\vec{R}$$

$$V^{+}_{\vec{K}',\vec{K}} = iU_{0}qC \frac{1}{\Omega} \int_{\Omega} e^{-i\vec{K}'\cdot\vec{R}} \, \mathrm{e}^{i\vec{Q}\cdot\vec{R}} \mathrm{e}^{i\vec{K}\cdot\vec{R}} \, d^{3}\vec{R}$$

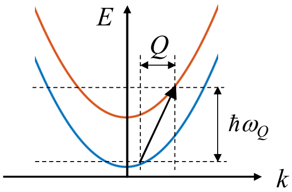
- Conservation of momentum

$$\vec{K}' = \vec{K} + \vec{Q} \rightarrow V^{+}_{\vec{K}',\vec{K}} = iU_{0}qC$$

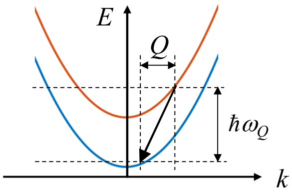
9

Scattering Rate

- **Absorption:** $W^{+}_{\vec{K}',\vec{K}} = \frac{2\pi}{\hbar} N_{\vec{Q}} U_{0}^{2} q^{2} C^{2} \delta_{\vec{K}',\vec{K}+\vec{Q}} \delta\left[\varepsilon\left(\vec{K}+\vec{Q}\right) - \varepsilon\left(\vec{K}\right) - \hbar\omega_{\vec{Q}} \right]$



- **Emission:** $W^{-}_{\vec{K}',\vec{K}} = \frac{2\pi}{\hbar} \left(N_{\vec{Q}} + 1 \right) U_{0}^{2} q^{2} C^{2} \delta_{\vec{K}',\vec{K}-\vec{Q}} \delta\left[\varepsilon\left(\vec{K}-\vec{Q}\right) - \varepsilon\left(\vec{K}\right) + \hbar\omega_{\vec{Q}} \right]$



10

Energy Transfer

- Typically:

$$\hbar\omega_{Q,\max} \approx 1\,\text{meV}$$

- Acoustic phonons do not carry away much energy
  $\rightarrow$ quasi-elastic approximation
      $\rightarrow$ energy change is neglected.