IMPURITY SCATTERING

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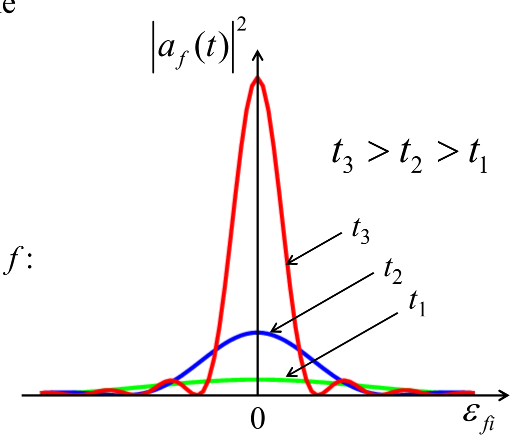
Fermi's Golden Rule

- The probability of finding the electron in the final state:

$$\left|a_f(t)\right|^2 = \frac{\left|V_{fi}\right|^2 t^2}{\hbar^2}\text{sinc}^2\left(\frac{\varepsilon_{fi}t}{2\hbar}\right)$$

- Transition rate from state i to f:

$$W_{fi} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \delta\left(\varepsilon_f - \varepsilon_i\right)$$



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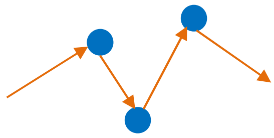
Impurity Scattering

- Important when phonons are few.
- The nature of the potential varies widely.
  - Charged impurities → long-range
  - Neutral impurities → short-range
- Alloy disorder and Interface-roughness scattering can be treated in a similar way.



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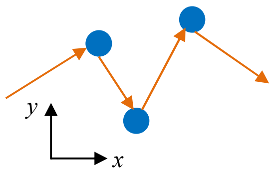
Theory

- Consider free electrons in two dimensions. Think about a system in a box of area A.

  Initial state: $\phi_i = A^{-1/2} e^{i\vec{k}\cdot\vec{r}}$

  Final state: $\phi_f = A^{-1/2} e^{i(\vec{k}+\vec{q})\cdot\vec{r}}$

  → Extra momentum $\hbar q$



- Matrix element due to perturbation:

$$V_{fi} = \int \phi_f^* \hat{V} \phi_i = \frac{1}{A}\int e^{-i(\vec{k}+\vec{q})\cdot\vec{r}}\, V(\vec{r}) e^{i\vec{k}\cdot\vec{r}}\, d^2\vec{r}$$

$$= \frac{1}{A}\int V(\vec{r}) e^{-i\vec{q}\cdot\vec{r}}\, d^2\vec{r} = A^{-1}\tilde{V}(\vec{q})$$

$\tilde{V}(\vec{q})$: 2-D Fourier transform of the scattering potential

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Fermi's Golden Rule

- The scattering rate from initial to final state:

$$W_{\vec{k}+\vec{q},\vec{k}} = \frac{1}{A^2}\frac{2\pi}{\hbar}\left|V(\vec{q})\right|^2 \delta\left[\varepsilon(\vec{k}+\vec{q}) - \varepsilon(\vec{k})\right]$$

- The effect of impurity diminishes as the system becomes larger!
- Total scattering rate for an electron:

$$\left(\frac{1}{\tau_i}\right)_{\text{Impurity}} = \sum_{\vec{q}} W_{\vec{k}+\vec{q},\vec{k}}$$

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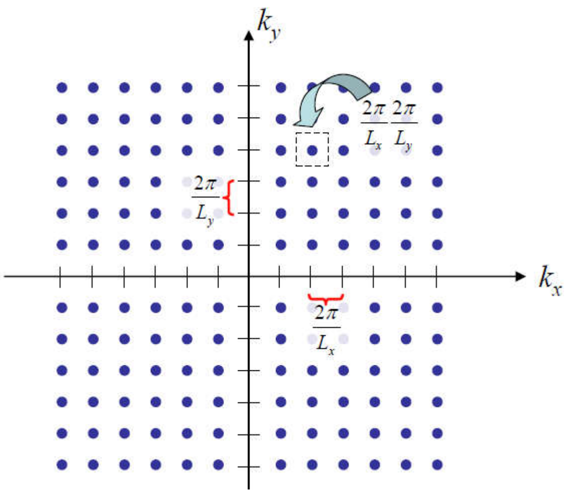
Integration Over k-Space

$$\left(\frac{1}{\tau_i}\right)_{\text{Impurity}} = \sum_{\vec{q}} W_{\vec{k}+\vec{q},\vec{k}}$$

- The sum over q can be converted into an integral

$$\sum_{\vec{q}} \rightarrow \frac{A}{(2\pi)^2}\int d^2\vec{q}$$



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Total Scattering Rate

- Total number of impurity in the system:

$$N_{\text{imp}}^{(2D)} = An_{\text{imp}}^{(2D)}$$

- Assume that scattering due to an impurity is independent of others.

$$\frac{1}{\tau_i} = \left[An_{\text{imp}}^{(2D)}\right]\frac{A}{(2\pi)^2}\int\frac{1}{A^2}\frac{2\pi}{\hbar}\left|\tilde{V}(\vec{q})\right|^2\delta\left[\varepsilon(\vec{k}+\vec{q})-\varepsilon(\vec{k})\right]d^2\vec{q}$$

$$= n_{\text{imp}}^{(2D)}\frac{2\pi}{\hbar}\int\left|\tilde{V}(\vec{q})\right|^2\delta\left[\varepsilon(\vec{k}+\vec{q})-\varepsilon(\vec{k})\right]\frac{d^2\vec{q}}{(2\pi)^2}$$

$\rightarrow$ The factors of A have vanished.

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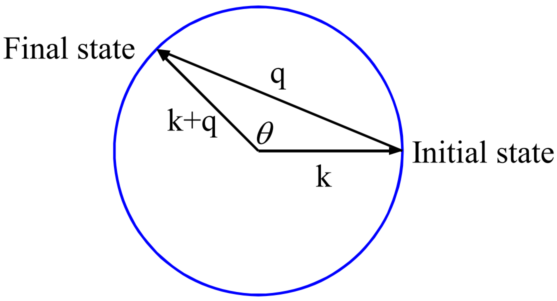
Transport Time

- $\mu = e\tau_{tr}/m \rightarrow \tau_{tr}$: Transport lifetime $\rightarrow$ different from τ_i
- The difference between τ_i and τ_{tr} lies in the weighting of different collisions.

$$\left|\vec{k}+\vec{q}\right| = \left|\vec{k}\right|$$

$$q = 2k\sin\frac{\theta}{2}$$



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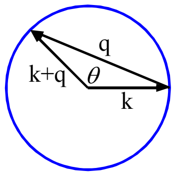
Weighting of Collisions

- The effect of a scattering depends on angle θ.
- Backscattering has a much larger effect on current.
- The contribution of collisions to transport is proportional to $1-\cos\theta$.

$$1-\cos\theta = \frac{q^2}{2k^2}$$



$$\frac{1}{\tau_i} = n_{\text{imp}}^{(2\text{D})} \frac{2\pi}{\hbar} \int \left|\tilde{V}(\vec{q})\right|^2 \delta\left[\varepsilon(\vec{k}+\vec{q}) - \varepsilon(\vec{k})\right] \frac{d^2\vec{q}}{(2\pi)^2}$$

$$\frac{1}{\tau_{\text{tr}}} = n_{\text{imp}}^{(2\text{D})} \frac{2\pi}{\hbar} \int \left(1-\cos\theta\right) \left|\tilde{V}(\vec{q})\right|^2 \delta\left[\varepsilon(\vec{k}+\vec{q}) - \varepsilon(\vec{k})\right] \frac{d^2\vec{q}}{(2\pi)^2}$$

$$= n_{\text{imp}}^{(2\text{D})} \frac{2\pi}{\hbar} \int \frac{q^2}{2k^2} \left|\tilde{V}(\vec{q})\right|^2 \delta\left[\varepsilon(\vec{k}+\vec{q}) - \varepsilon(\vec{k})\right] \frac{d^2\vec{q}}{(2\pi)^2}$$

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GOLDEN RULE FOR OSCILLATING POTENTIALS

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Oscillating Potential

- If the perturbation varies harmonically

$$V(t) = 2\tilde{V}\cos\omega_0 t = \tilde{V}\left(e^{-i\omega_0 t} + e^{i\omega_0 t}\right)$$

- The matrix element changes to

$$V_{fi}(t) = 2V_{fi}\cos\omega_0 t = V_{fi}\left(e^{-i\omega_0 t} + e^{i\omega_0 t}\right)$$

- Remember the scattering probability amplitude

$$a_f(t) = \frac{1}{i\hbar}\int_0^t V_{fi}(t')\, e^{i\varepsilon_{fi}t'/\hbar}\, dt'$$

- Therefore

$$a_f(t) = \frac{1}{i\hbar}V_{fi}\int_0^t \left(e^{-i\omega_0 t'} + e^{i\omega_0 t'}\right) e^{i\varepsilon_{fi}t'/\hbar}\, dt'$$

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Oscillating Potential

- The probability of being found in the final state:

$$\left|a_f(t)\right|^2 = \frac{\left|V_{fi}\right|^2 t^2}{\hbar^2}\left[\operatorname{sinc}^2\frac{\left(\varepsilon_{fi} - \hbar\omega_0\right)t}{2\hbar} + \operatorname{sinc}^2\frac{\left(\varepsilon_{fi} + \hbar\omega_0\right)t}{2\hbar}\right]$$
$$+ \frac{\left|V_{fi}\right|^2 t^2}{\hbar^2}\left[2\cos\omega_0 t\,\operatorname{sinc}\frac{\left(\varepsilon_{fi} - \hbar\omega_0\right)t}{2\hbar}\operatorname{sinc}\frac{\left(\varepsilon_{fi} + \hbar\omega_0\right)t}{2\hbar}\right]$$

- The first term is centered on $\varepsilon_{fi} = \hbar\omega_0,\ \varepsilon_f = \varepsilon_i + \hbar\omega_0$
- The second term is centered on $\varepsilon_{fi} = -\hbar\omega_0,\ \varepsilon_f = \varepsilon_i - \hbar\omega_0$
- The third term can be ignored when $\omega_0 t \gg 1$

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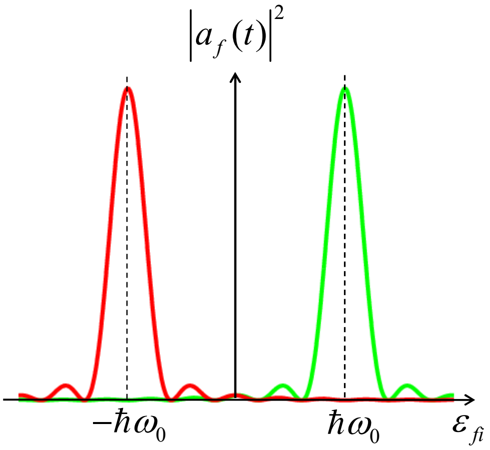
Oscillating Potential

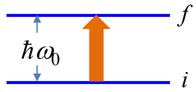


Transition Rate

- Fermi's golden rule for a harmonic perturbation:

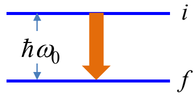
Absorption:

$$W_{fi} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \delta\left(\varepsilon_f - \varepsilon_i - \hbar\omega_0\right)$$



Emission:

$$W_{fi} = \frac{2\pi}{\hbar}\left|V_{fi}\right|^2 \delta\left(\varepsilon_f - \varepsilon_i + \hbar\omega_0\right)$$



Class Test – 3

Day: 15 January 2019

Syllabus: Lectures 17 – 18