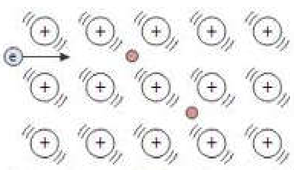
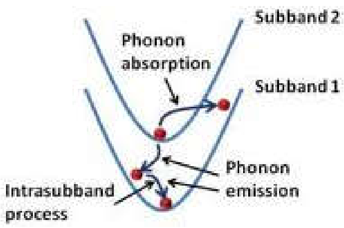
SCATTERING RATES

Electron Scattering

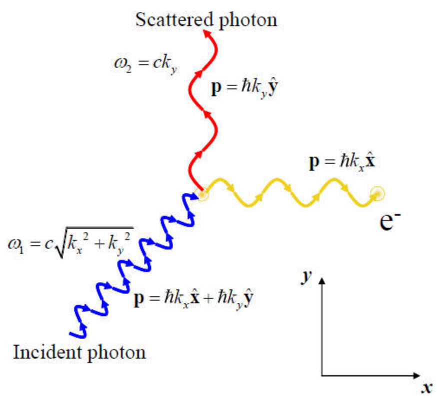
Electron-impurity interaction



Electron-Phonon Interaction



Electron-Photon Interaction



Scattering Processes

- Two broad classes of scattering processes:
  - Potentials that are constant in time, such as impurities in a crystal, which do not change the energy of the particle being scattered → Elastic scattering.
  - Potentials that vary harmonically in time as $\cos\omega_q t$, such as phonons and photons, which change the energy of the particle by $\pm\hbar\omega_q$ → Inelastic scattering.
- We need to calculate the rates at which particles are scattered from one state to another by a perturbation → Fermi's Golden Rule

3

Constant Perturbation

- We divide the Hamiltonian into a large unperturbed part and a small perturbation that is turned on at $t = 0$.

$$\hat{H} = \hat{H}_0 + \hat{V}(t)$$

- At $t < 0$, the electron is in an initial state i:

$$\Psi(t) = \Phi_i(t) = \phi_i e^{-i\varepsilon_i t/\hbar}$$

- For $t > 0$: $\hat{H}\Psi(t) = \left[\hat{H}_0 + \hat{V}(t)\right]\Psi(t) = i\hbar\frac{\partial}{\partial t}\Psi(t)$

→ Time-dependent Schrodinger equation.

4

Solution

- Expand the exact solution :

$$\Psi(t) = \sum_j a_j(t)\Phi_j(t)$$

- $a_j(t)$: probability amplitude for being in state j at time t, with initial value $a_j(t = 0) = \delta_{ij}$.

$$\left[\hat{H}_0 + \hat{V}(t)\right]\sum_j a_j(t)\Phi_j(t) = i\hbar\frac{\partial}{\partial t}\sum_j a_j(t)\Phi_j(t)$$

$$\sum_j a_j(t)\hat{H}_0\Phi_j(t) + \sum_j a_j(t)\hat{V}(t)\Phi_j(t)$$

$$= i\hbar\sum_j a_j(t)\frac{\partial\Phi_j(t)}{\partial t} + i\hbar\sum_j\frac{\partial a_j(t)}{\partial t}\Phi_j(t)$$

5

Solution

- The first term on each side cancels.

$$i\hbar\sum_j\frac{\partial a_j(t)}{\partial t}\phi_j e^{-i\varepsilon_j t/\hbar} = \sum_j a_j(t)\hat{V}(t)\phi_j e^{-i\varepsilon_j t/\hbar}$$

- Multiply throughout by ϕ_f* and integrate over space.

$$i\hbar\frac{da_f(t)}{dt}e^{-i\varepsilon_f t/\hbar} = \sum_j a_j(t)e^{-i\varepsilon_j t/\hbar}\int\phi_f^*\hat{V}\phi_j$$

$$= \sum_j a_j(t)e^{-i\varepsilon_j t/\hbar}V_{fj}(t)$$

$$\frac{da_f(t)}{dt} = \frac{1}{i\hbar}\sum_j a_j(t)V_{fj}(t)e^{i\varepsilon_{fj}t/\hbar}$$

$$\varepsilon_{fj} = \varepsilon_f - \varepsilon_j$$

6

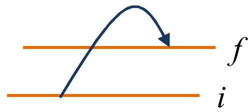
Probability

$$\frac{da_f(t)}{dt} = \frac{1}{i\hbar}\sum_j a_j(t) V_{fj}(t) e^{i\varepsilon_{fj}t/\hbar}$$

- Let's assume that that before scattering there is only φ_i

$$a_j(t) = \delta_{ij}$$

$$\frac{da_f(t)}{dt} = \frac{1}{i\hbar} V_{fi}(t) e^{i\varepsilon_{fi}t/\hbar}$$



- This can be integrated for $f \neq i$:

$$a_f(t) = \frac{1}{i\hbar}\int_0^t V_{fi}(t') e^{i\varepsilon_{fi}t'/\hbar} dt'$$

- This is the probability amplitude of state f at time t.

Fermi's Golden Rule

$$a_f(t) = \frac{1}{i\hbar}\int_0^t V_{fi}(t') e^{i\varepsilon_{fi}t'/\hbar} dt'$$

- ***Assume constant perturbation:*** V_{fi} can be pulled out of the integral

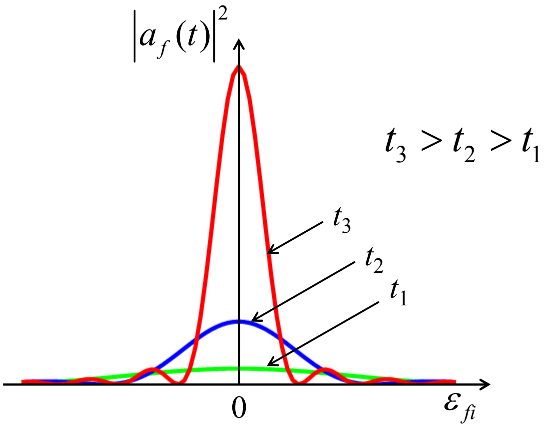
$$a_f(t) = -V_{fi}\frac{e^{i\varepsilon_{fi}t/\hbar} - 1}{\varepsilon_{fi}} = -V_{fi} e^{i\varepsilon_{fi}t/2\hbar}\frac{e^{i\varepsilon_{fi}t/2\hbar} - e^{-i\varepsilon_{fi}t/2\hbar}}{\varepsilon_{fi}}$$

$$= -i V_{fi} e^{i\varepsilon_{fi}t/2\hbar}\frac{\sin\left(\varepsilon_{fi}t/2\hbar\right)}{\varepsilon_{fi}/2} = -iV_{fi}\frac{t}{\hbar}e^{i\varepsilon_{fi}t/2\hbar}\,\mathrm{sinc}\left(\frac{\varepsilon_{fi}t}{2\hbar}\right)$$

- The probability of finding the electron in the final state:

$$\left|a_f(t)\right|^2 = \left|V_{fi}\right|^2\left[\frac{\sin\left(\varepsilon_{fi}t/2\hbar\right)}{\varepsilon_{fi}/2}\right]^2 = \frac{\left|V_{fi}\right|^2 t^2}{\hbar^2}\mathrm{sinc}^2\left(\frac{\varepsilon_{fi}t}{2\hbar}\right)$$

Fermi's Golden Rule



$$|a_f(t)|^2 = \frac{|V_{fi}|^2 t^2}{\hbar^2}\text{sinc}^2\left(\frac{\varepsilon_{fi}t}{2\hbar}\right)$$

Transition Rate

$$\int_{-\infty}^{\infty}\text{sinc}^2 x\,dx = \pi$$

$$\int_{-\infty}^{\infty} t^2\text{sinc}^2\left(\varepsilon_{fi}t/2\hbar\right)d\varepsilon_{fi} = 2\pi\hbar t$$

$$t^2\text{sinc}^2\left(\varepsilon_{fi}t/2\hbar\right)\to 2\pi\hbar t\delta\left(\varepsilon_{fi}\right)$$

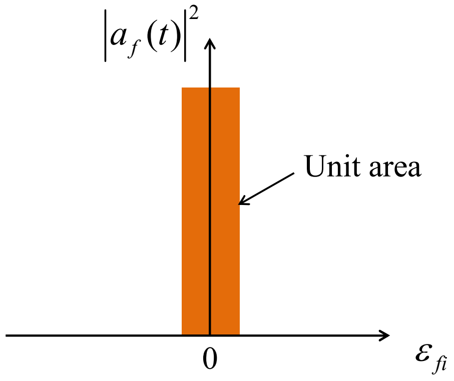
- The probability of being found in a final state when $t \to \infty$

$$|a_f(t)|^2 \approx \frac{2\pi}{\hbar}|V_{fi}|^2\,\delta\left(\varepsilon_{fi}\right)t$$

- Transition rate from state i to f:

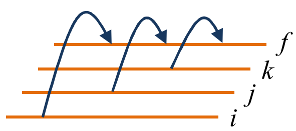
$$W_{fi} = \frac{2\pi}{\hbar}|V_{fi}|^2\,\delta\left(\varepsilon_f - \varepsilon_i\right)$$

Collisional Broadening



11

Multiple Initial States



$$W_f = \frac{2\pi}{\hbar} \sum_i |V_{fi}|^2 \delta(\varepsilon_f - \varepsilon_i)$$

12