LASER AMPLIFICATION

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Laser Amplification and Gain

- Now we have the means to build a laser amplifier.
- Strong (usually) electric dipole transitions.
- Pumping methods for achieving population inversion.
- Now we have a means of generating power per unit volume by stimulated emission.

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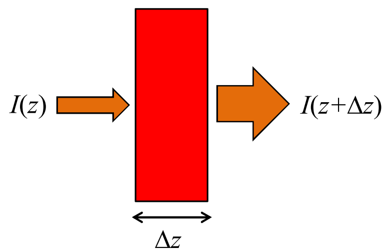
Gain, Absorption

Let us consider a plane wave going through a medium. We will see that power added to or absorbed from the field.



I : Intensity

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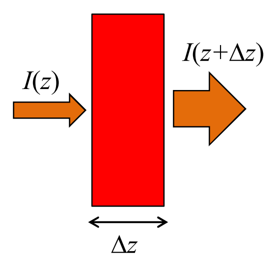
Rate of Change with Distance

Change in power

$$\Delta P = A\{I(z+\Delta z) - I(z)\} = A\Delta z 2gI$$

$$A\frac{dI}{dz}\Delta z = A\Delta z 2gI, \qquad 2g \propto K, n \propto I$$

$$\frac{dI}{dz} = 2gI \Rightarrow I = I_0 e^{2gz}$$



$2g$: Gain coefficient

We have seen that $2g$ is proportional to $(N_2 - N_1)$. Usually when we pass light through a medium it is absorbed since the atoms are in the ground state, i.e., $N_1 >> N_2$ or $2g < 0$. Note if we can invert the population by making $N_2 > N_1$ then $2g > 0$ and we get exponential gain rather than loss!!

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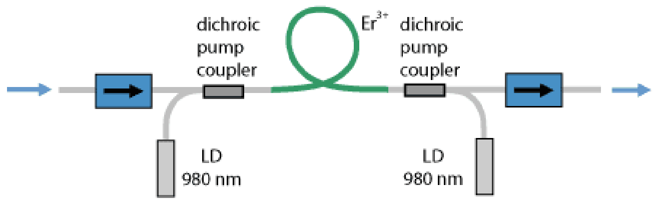
Laser Amplifiers



Thus if we have population inversion, we have power per unit volume generated and added in phase with the incoming field. This process allows us to amplify weak optical signals. In optical fibers, the amplifiers are typically made from Erbium ions doped into the fiber. External laser light (at a wavelength shorter than the signal's) is coupled into the fiber to optically pump the erbium ions. These amplifiers are spaced typically somewhere between 25 to 100 km apart. At the operational wavelength near 1550 nm modern optical fiber has a loss of 0.2 dB/km yielding about 20 dB of loss in 100 km. The optical amplifier then overcomes this loss to bring the signal back up to its initial level. Then the signal propagates another distance and is then re-amplified. Note that trans-oceanic optical communications work exactly this way with amplifiers spaced about 50 km apart sitting on the bottom of the ocean!! Commercially as many as 64 channels at 10 Gb/s can operate on a signal fiber on these links (much more is possible)!!

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Noise

Of course there is a problem introduced by repeated amplification. In addition to the stimulated emission there is spontaneous emission which is random. The fraction of the spontaneous emission that is in the direction (same mode as) of the signal will then show up as noise. We can reduce some of the noise by optically filtering it out but some remains at the same wavelength as the signal. Fortunately the extra noise added per amplifier is small and thus one can use many amplifiers in cascade. However, the noise does accumulate.

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Single Frequency Fields

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + j\omega \vec{D}$$

$$\vec{J} = \sigma \vec{E}$$

$$\begin{aligned}\vec{D}(\omega) &= \varepsilon_b(\omega)\vec{E}(\omega) + \vec{P}_a(\omega) \\ &= \varepsilon_b(\omega)\vec{E}(\omega) + \varepsilon_0 \chi_a(\omega)\vec{E}(\omega) \\ &= \varepsilon_b(\omega)\left(1 + \frac{\varepsilon_0}{\varepsilon_b(\omega)}\chi_a(\omega)\right)\vec{E}(\omega)\end{aligned}$$

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Review EM

Here we will use plane waves with propagation in one dimension, the *z* direction.

$$\nabla \times \nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{B}}{\partial t} = -j\omega \nabla \times \vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + j\omega \vec{D}$$

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

Now if the dielectric constant does not vary with distance then $\nabla \cdot \vec{E} = 0$.

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Wave Equation

$$-\nabla^2\vec{E} = -j\omega\nabla\times\vec{B} = -j\omega\mu\nabla\times\vec{H}$$
$$= -j\omega\mu\left(\vec{J} + j\omega\vec{D}\right)$$
$$= -j\omega\mu\left[\sigma\vec{E} + j\omega\varepsilon_b\left(1+\frac{\varepsilon_0}{\varepsilon_b}\chi_a\right)\vec{E}\right]$$
$$= \omega^2\mu\varepsilon_b\left[1+\frac{\varepsilon_0}{\varepsilon_b}\chi_a - j\frac{\sigma}{\omega\varepsilon_b}\right]\vec{E}$$

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One Dimension

Using the plane wave approximation and assuming both the background material and the atoms leading to gain provide isotropic response we can reduce this to a simple one dimensional equation. Also we can then use linear polarized light without loss of generality.

$$-\frac{\partial^2 E}{\partial z^2} = \omega^2\mu\varepsilon_b\left[1+\frac{\varepsilon_0}{\varepsilon_b}\chi_a - j\frac{\sigma}{\omega\varepsilon_b}\right]E(z)$$
$$\frac{\partial^2 E}{\partial z^2} + \omega^2\mu\varepsilon_b\left[1+\frac{\varepsilon_0}{\varepsilon_b}\chi_a - j\frac{\sigma}{\omega\varepsilon_b}\right]E(z) = 0$$

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$\sigma = 0, \quad \chi = 0$

If we have no conductivity and no susceptibility from the atoms, i.e., just a lossless background dielectric

$$\frac{\partial^2 E}{\partial z^2} + \omega^2 \mu \varepsilon_b E(z) = 0$$

$$E = E_0 e^{\pm jkz} \Rightarrow -k^2 + \omega^2 \mu \varepsilon_b = 0$$

$$k = \sqrt{\mu \varepsilon_b}\,\omega = n\frac{\omega}{c}, n = \sqrt{\frac{\mu \varepsilon_b}{\mu_0 \varepsilon_0}}$$

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$\sigma \neq 0, \quad \chi \neq 0$

The conductivity and susceptibility will in general be small so we can approximate

$$\frac{\partial^2 E}{\partial z^2} + \omega^2 \mu \varepsilon_b \left[1 + \frac{\varepsilon_0}{\varepsilon_b}\chi - j\frac{\sigma}{\omega \varepsilon_b}\right] E(z) = 0$$

$$E = E_0 e^{\pm jkz} \Rightarrow k^2 = \omega^2 \mu \varepsilon_b \left[1 + \frac{\varepsilon_0}{\varepsilon_b}\chi - j\frac{\sigma}{\omega \varepsilon_b}\right]$$

$$\frac{\varepsilon_0}{\varepsilon_b}|\chi| \ll 1, \frac{|\sigma|}{\omega \varepsilon_b} \ll 1, k \approx n\frac{\omega}{c}\left[1 + \frac{\varepsilon_0}{2\varepsilon_b}\chi - j\frac{\sigma}{2\omega \varepsilon_b}\right], \chi = \chi' + j\chi''$$

$$k \approx n\frac{\omega}{c}\left[1 + \frac{\varepsilon_0}{2\varepsilon_b}(\chi' + j\chi'') - j\frac{\sigma}{2\omega \varepsilon_b}\right]$$

$$= n\frac{\omega}{c}\left(1 + \frac{\varepsilon_0}{2\varepsilon_b}\chi'\right) - jn\frac{\omega}{c}\left(\frac{\sigma}{2\omega \varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b}\chi''\right)$$

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Loss

Now let's take propagation for simplicity in the $+z$ direction so our propagating wave is of the form $e^{j(\omega t - kz)}$

$$k = n\frac{\omega}{c}\left(1+\frac{\varepsilon_0}{2\varepsilon_b}\chi'\right) - jn\frac{\omega}{c}\left(\frac{\sigma}{2\omega\varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b}\chi''\right)$$

$$e^{j(\omega t - kz)} = e^{j\left[\omega t - n\frac{\omega}{c}\left(1+\frac{\varepsilon_0}{2\varepsilon_b}\chi'\right)z\right]} e^{-n\frac{\omega}{c}\left(\frac{\sigma}{2\omega\varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b}\chi''\right)z}$$

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Additional Phase

Now we see that the propagation gets an additional phase shift as a function of propagation distance z.

$$\delta\phi = n\frac{\omega}{c}\left(\frac{\varepsilon_0}{2\varepsilon_b}\chi'\right)z$$

$$\frac{\delta\phi}{\phi} = \frac{n\frac{\omega}{c}\left(\frac{\varepsilon_0}{2\varepsilon_b}\chi'\right)z}{n\frac{\omega}{c}z} = \frac{\varepsilon_0}{2\varepsilon_b}\chi'$$

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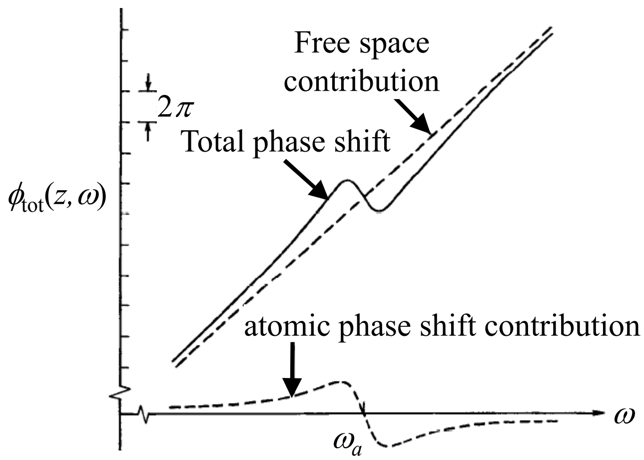
Change Near Resonance



$$\chi' = \frac{3}{4\pi^2}\frac{(N_2 - N_1)\lambda^3\gamma_{rad}}{\Delta\omega_a}\frac{2(\omega-\omega_a)/\Delta\omega_a}{1+\left[2(\omega-\omega_a)/\Delta\omega_a\right]^2}$$

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Gain or Loss

The term in the exponent with the conductivity represents loss. Assume that $\chi = 0$ then it is obvious that we have loss and the loss per unit length is $n\sigma/(2c\varepsilon)$ for the field and $n\sigma/(c\varepsilon)$ for the power.

$$E(z) = E_0 e^{-n\frac{\omega}{c}\left[\frac{\sigma}{2\omega\varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b(\omega)}\chi''\right]z}$$

$$\sigma = 0 \Rightarrow E(z) = E_0 e^{n\frac{\omega}{c}\frac{\varepsilon_0\chi''}{2\varepsilon_b(\omega)}z}$$

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Gain

Suppose we have a medium with gain. Then let's look at the field and power gain after a certain length L of propagation.

$$\tilde{g}(\omega) = \frac{\tilde{E}(L)}{\tilde{E}(0)}$$

$$G(\omega) = \frac{I(L)}{I(0)} = |\tilde{g}(\omega)|^2 = e^{[2\alpha_m(\omega) - 2\alpha_0]L}$$

$$\alpha_m(\omega) = n\frac{\omega\varepsilon_0}{2c\varepsilon_b}\chi''(\omega) = n\frac{\omega\varepsilon_0}{2c\varepsilon_b}\frac{\chi_0''}{1+\left[2\frac{\omega-\omega_a}{\Delta\omega_a}\right]^2}$$

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Gain Lineshape

$$G(\omega) = e^{\left[n\frac{\omega\varepsilon_0 L\chi_0''}{c\varepsilon_b}\frac{1}{1+\left[2(\omega-\omega_a)/\Delta\omega_a\right]^2}\right]}$$

- Note that the lorentzian lineshape appears in the exponent.
- The exponential gain falls off much more rapidly with detuning than the atomic lineshape itself → gain narrowing.

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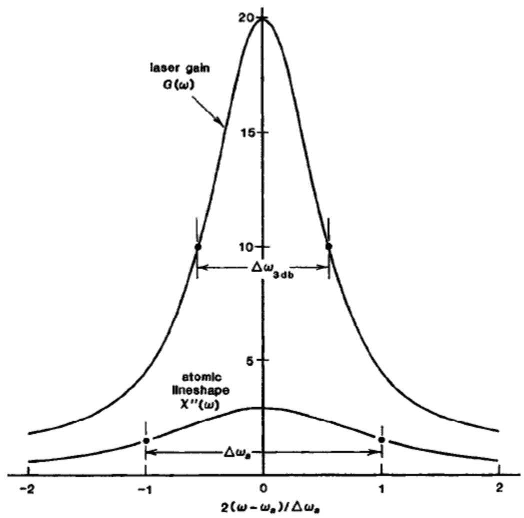
Gain Narrowing



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Bandwidth

$$G_{\mathrm{dB}}(\omega)=\frac{G_{\mathrm{dB}}(\omega_a)}{1+\left[2\left(\omega-\omega_a\right)/\Delta\omega_a\right]^2}=G_{\mathrm{dB}}(\omega_a)-3$$

$$\left(\omega-\omega_a\right)_{3\mathrm{dB}}=\pm\frac{\Delta\omega_a}{2}\sqrt{\frac{3}{G_{\mathrm{dB}}(\omega_a)-3}}$$

Full 3-dB bandwidth

$$\Delta\omega_{3\mathrm{dB}}=\Delta\omega_a\sqrt{\frac{3}{G_{\mathrm{dB}}(\omega_a)-3}}$$

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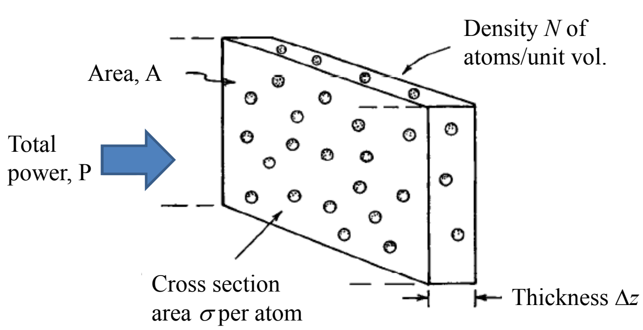
TRANSITION CROSS SECTIONS

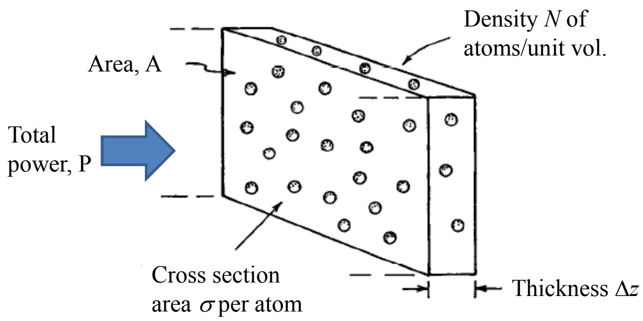
Concept



The net power ΔP_{abs} absorbed

$$\Delta P_{abs} = \sigma \times \frac{P}{A} = \sigma I$$

Net Power Absorbed



Total effective absorbing area $= N_1\sigma_{12}A\Delta z$

Total effective emitting area $= N_2\sigma_{21}A\Delta z$

$$\Delta P_{\text{abs}} = N_1\sigma_{12}A\Delta z \times \frac{P}{A} - N_2\sigma_{21}A\Delta z \times \frac{P}{A}$$

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Cross Sections ↔ Amplification Coefficients

The net growth or decay rate

$$\frac{dP}{dz} = -\lim_{\Delta z \to 0}\left(\frac{\Delta P_{\text{abs}}}{\Delta z}\right) = -\left(N_1\sigma_{12} - N_2\sigma_{21}\right)P$$

$$\frac{1}{P}\frac{dP}{dz} = -\left(N_1\sigma_{12} - N_2\sigma_{21}\right) = -\left(\frac{g_2}{g_1}N_1 - N_2\right)\sigma_{21}$$

$$\frac{1}{I}\frac{dI}{dz} = -\Delta N_{12}\sigma_{21}$$

$$\frac{1}{I}\frac{dI}{dz} = -2\alpha_m$$

$$2\alpha_m = \Delta N_{12}\sigma_{21}$$

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Transition Cross Section

$$2\alpha_m(\omega) = \Delta N \sigma(\omega) = \frac{2\pi}{\lambda}\chi''(\omega)$$

$$= \frac{3^*}{2\pi\lambda}\frac{\Delta N \lambda^3 \gamma_{\text{rad}}}{\Delta\omega_a}\frac{1}{1+\left[2\dfrac{\omega-\omega_a}{\omega_a}\right]^2}$$

$$\sigma(\omega) = \frac{3^*}{2\pi}\frac{\lambda^2 \gamma_{\text{rad}}}{\Delta\omega_a}\frac{1}{1+\left[2\dfrac{\omega-\omega_a}{\omega_a}\right]^2}$$

$$\sigma(\omega_a) = \frac{3^*}{2\pi}\frac{\lambda^2 \gamma_{\text{rad}}}{\Delta\omega_a}$$

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Maximum Value

- The cross section will be maximum for a transition that has purely radiative lifetime broadening only, and no other line-broadening effects, so that $\Delta\omega_a = \gamma_{\text{rad}}$.
- If the atoms all have their transition axes aligned and the incident fields are optimally polarized, so that $3^* = 3$.

$$\sigma_{\max} = \frac{3\lambda^2}{2\pi}$$

Effective absorption cross-section can be many times larger than the actual cross-section of the atoms!!!

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HOMOGENEOUS SATURATION

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Gain Saturation

- For a strong enough input signal the stimulated transition rate may become large enough to saturate the population difference, and thus reduce the gain coefficient seen by the signal. This process is commonly referred to as saturation of the gain (or absorption) coefficient by the applied signal.
- Saturation behavior in a laser amplifier can be expected therefore whenever the signal strength becomes strong enough for the signal itself to reduce the signal growth or attenuation rate.

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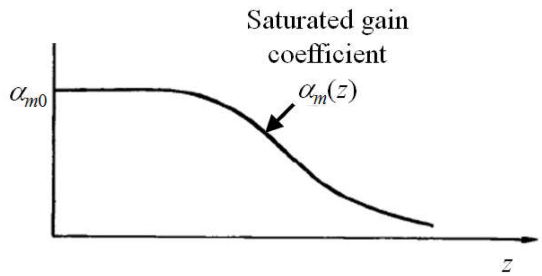
Homogeneous Saturation



The growth rate for the signal intensity

$$\frac{1}{I(z)}\frac{dI(z)}{dz} = 2\alpha_m(I) = \frac{2\alpha_{m0}}{1+\dfrac{I(z)}{I_{\text{sat}}}}$$

α_{m0}: Unsaturated gain coefficient

I_{sat}: Saturation intensity

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Integration

$$\int_{I=I_{\text{in}}}^{I=I_{\text{out}}}\left[\frac{1}{I}+\frac{1}{I_{\text{sat}}}\right]dI = 2\alpha_{m0}\int_{z=0}^{z=L} dz$$

$$\ln\left(\frac{I_{\text{out}}}{I_{\text{in}}}\right)+\frac{I_{\text{out}}-I_{\text{in}}}{I_{\text{sat}}} = 2\alpha_{m0}L = \ln G_0$$

$G_0 = \exp\left(2\alpha_{m0}L\right) \rightarrow$ small-signal or unsaturated power gain

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Power Gain

Overall power gain

$$G = \frac{I_{\text{out}}}{I_{\text{in}}} = G_0 \exp\left[-\frac{I_{\text{out}} - I_{\text{in}}}{I_{\text{sat}}}\right]$$

Written in different forms

$$G = \frac{I_{\text{out}}}{I_{\text{in}}} = G_0 \exp\left[-\frac{(G-1) I_{\text{in}}}{I_{\text{sat}}}\right] = G_0 \exp\left[-\frac{(G-1) I_{\text{out}}}{G I_{\text{sat}}}\right]$$

$$\frac{I_{\text{in}}}{I_{\text{sat}}} = \frac{1}{G-1} \ln\left(\frac{G_0}{G}\right)$$

$$\frac{I_{\text{out}}}{I_{\text{sat}}} = \frac{G}{G-1} \ln\left(\frac{G_0}{G}\right)$$

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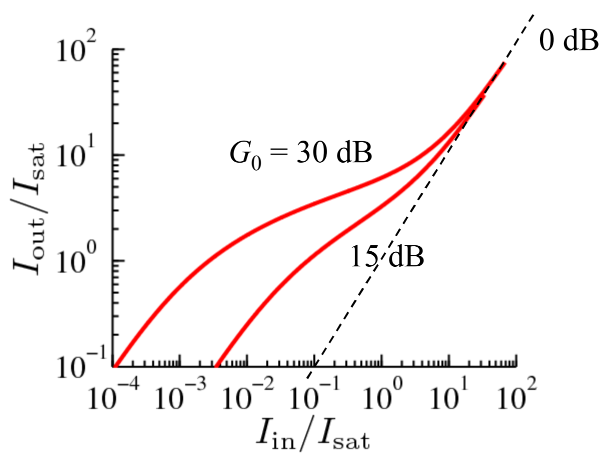
Output Vs. Input



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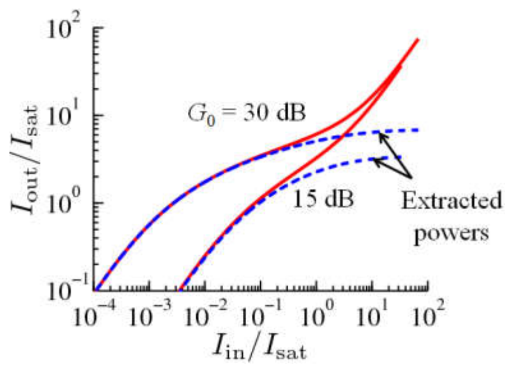
Extracted Power

Power per unit area extracted from the amplifier

$$I_{\text{extr}} = I_{\text{out}} - I_{\text{in}} = \ln\left(\frac{G_0}{G}\right) \times I_{\text{sat}}$$

For low input intensity and high gain, the output power and the extracted power are essentially the same.



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