GAIN SATURATION

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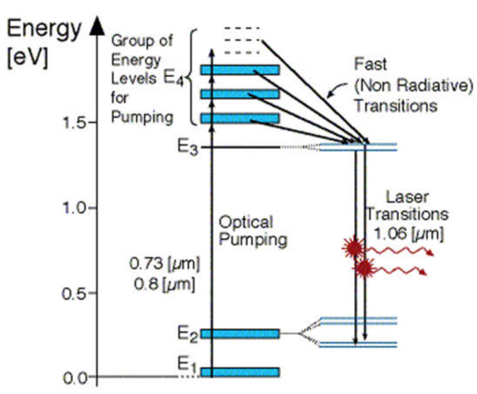
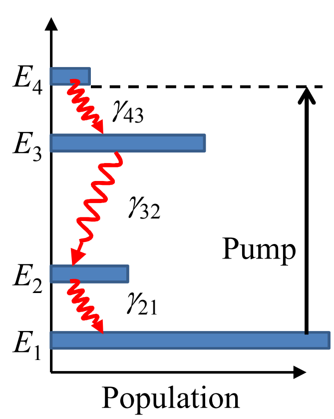
Nd:YAG Lasers

- Energy diagram of Nd:YAG laser system





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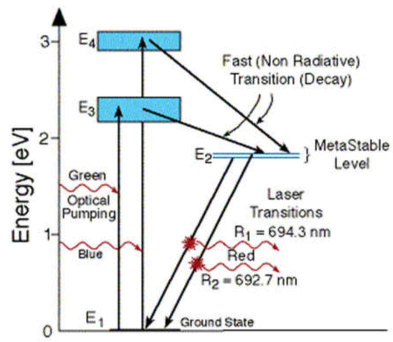
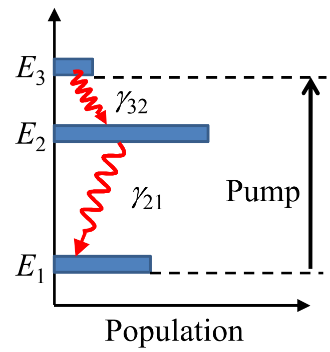
Ruby Laser

- Energy diagram of Ruby laser:





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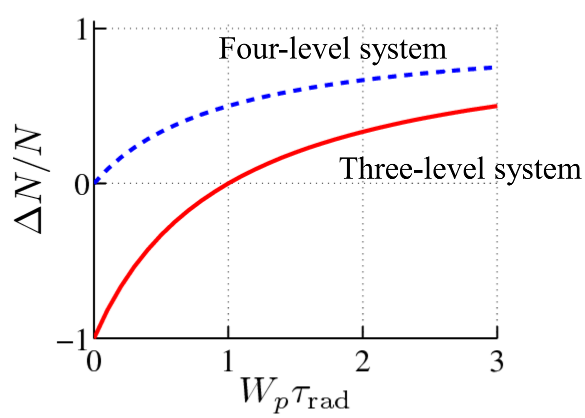
Comparison



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Gain Saturation

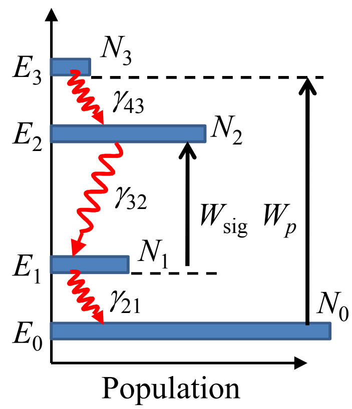
$$W_{03} = W_{30} = W_p$$

$$\frac{dN_3}{dt} = W_p\left(N_0 - N_3\right) \approx W_p N_0$$

$$N_0 \approx N$$

In this situation, $3 \rightarrow 0$ pumping can be neglected, so

$$W_p N_0 \approx W_p N.$$



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Rate Equations

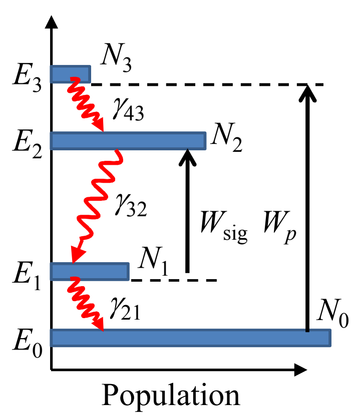
$$R_p = \eta_p W_p N_0$$

$$W_{12} = W_{21} = W_{sig}$$

$$\frac{dN_2}{dt} = R_p - W_{sig}\left(N_2 - N_1\right) - \gamma_2 N_2$$

$$\frac{dN_1}{dt} = W_{sig}\left(N_2 - N_1\right) + \gamma_{21} N_2 - \gamma_1 N_1$$

$$\gamma_2 = \gamma_{21} + \gamma_{20}$$



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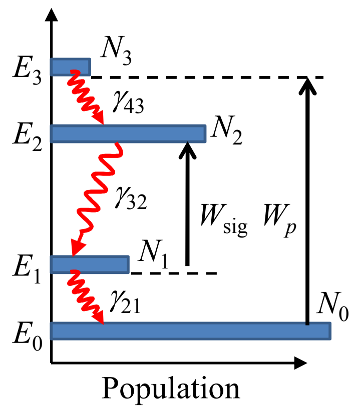
Steady-State Solutions

$$N_1 = \frac{W_{sig} + \gamma_{21}}{W_{sig}(\gamma_1 + \gamma_{20}) + \gamma_1\gamma_2} R_p$$

$$N_2 = \frac{W_{sig} + \gamma_1}{W_{sig}(\gamma_1 + \gamma_{20}) + \gamma_1\gamma_2} R_p$$

Note:

- $N_1 + N_2$ does not remain constant.



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Saturation Behavior

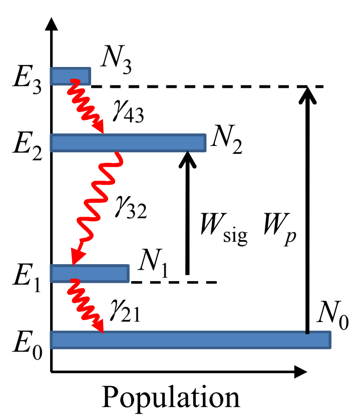
$$\Delta N_{21} = N_2 - N_1 = \left(\frac{\gamma_1 - \gamma_{21}}{\gamma_1\gamma_2}\right) \times \frac{R_p}{1 + \left(\frac{\gamma_1 + \gamma_{20}}{\gamma_1\gamma_2}\right) W_{sig}}$$

$$\Delta N_{21} = \Delta N_0 \frac{1}{1 + W_{sig}\tau_{eff}}$$

$$\Delta N_0 = \frac{\gamma_1 - \gamma_{21}}{\gamma_1\gamma_2} R_p = \left(1 - \frac{\tau_1}{\tau_{21}}\right)\tau_2 R_p$$

$$\frac{1}{\tau_{eff}} = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_{20}} \Rightarrow \tau_{eff} = \tau_2\left(1 + \frac{\tau_1}{\tau_{20}}\right)$$

τ_{eff}: effective recovery time of the gain



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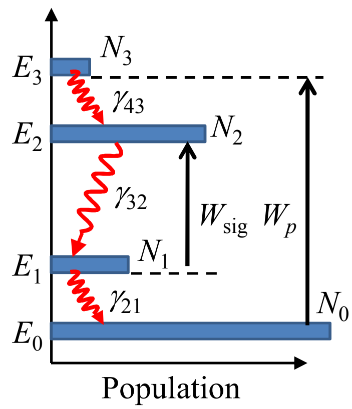
Saturation Behavior

If the upper laser level relaxes mainly into the lower laser level and not directly to the ground level,

$$\tau_{\text{eff}} = \tau_2\left(1+\frac{\tau_1}{\tau_{20}}\right)\approx \tau_2$$

$$\Delta N_{21} \approx R_p\left(\tau_2-\tau_1\right)\frac{1}{1+W_{\text{sig}}\tau_2}$$



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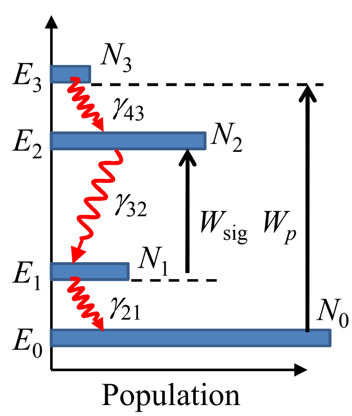
- In a two-level system

$$\Delta N = \Delta N_0\frac{1}{1+2W_{12}T_1}$$

- In a system that we discussed here

$$\Delta N_{21} = \Delta N_0\frac{1}{1+W_{\text{sig}}\tau_2}$$



TRANSIENT LASER PUMPING

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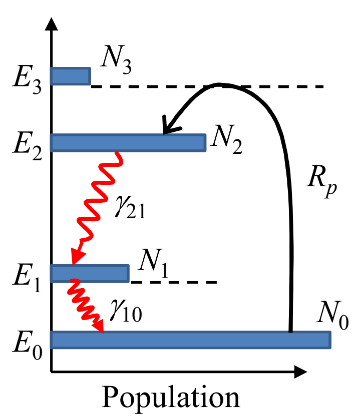
Transient

- Let us take R_p as a function of time → $R_p(t) = \eta_p W_p(t) N_0$
- Let us assume: $W_{sig} = 0$.



$$\frac{dN_2(t)}{dt} = R_p(t) - \gamma_2 N_2(t)$$

$$N_2(t) = \int_{-\infty}^{t} R_p(t') e^{-\gamma_2 (t-t')} dt'$$

- Thus the system relaxes on a time scale of γ_2.

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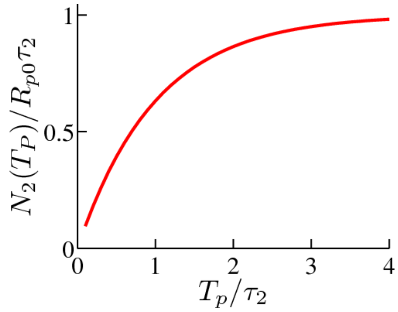
Square Pump Pulse

- Consider a square pump pulse with constant amplitude R_{p0} and duration T_p.
- What is the upper level population at the end of the pulse?

$$N_2\left(T_p\right) = \int_0^{T_p} R_p(t') e^{-\gamma_2\left(T_p - t'\right)} dt'$$

$$= R_{p0} \frac{e^{-\gamma_2 T_p}}{\gamma_2}\left[e^{\gamma_2 T_p} - 1\right]$$

$$= R_{p0}\tau_2 \left[1 - e^{-\frac{T_p}{\tau_2}}\right]$$



- Note that it is of little use to increase T_p beyond $2\tau_2$

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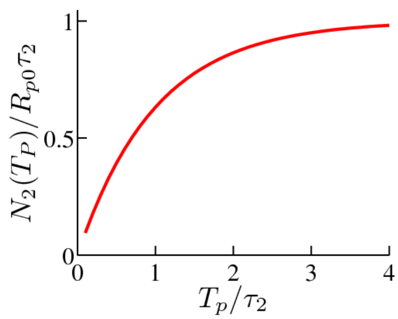
Pumping Efficiency

Ratio of the maximum number of atoms stored in the upper level, just at the end of the pumping pulse, to the total number of pump photons sent in or atoms lifted up during the pump pulse.

$$\eta_p = \frac{N_2\left(t = T_p\right)}{R_{p0} T_p} = \frac{1 - e^{-\frac{T_p}{\tau_2}}}{\frac{T_p}{\tau_2}}$$



- $\eta_p \approx 63\%$ when $T_p/\tau_2 \approx 1$.
- $\eta_p \approx 90\%$ when $T_p/\tau_2 \approx 0.2$.

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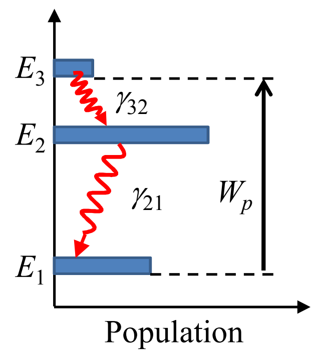
Transient Rate-Equation

Let us assume that γ_{32} is so fast that all the atoms pumped into level 3 may be assumed to relax instantaneously into level 2.

$$N_3 \approx 0$$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt} \approx -W_p(t)N_1(t) + \frac{N_2(t)}{\tau}$$

$$N_1(t) + N_2(t) + N_3(t) \approx N_1(t) + N_2(t) \approx N$$



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Pulsed Inversion

Suppose a square pump pulse with constant pump intensity W_p is turned on at $t = 0$.

$$\Delta N = N_2 - N_1$$

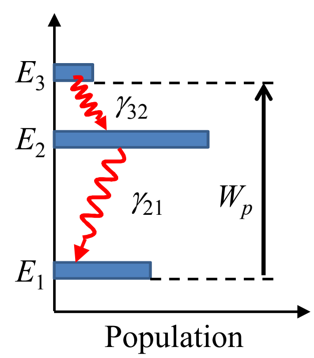
$$N_1 = \frac{N - \Delta N}{2}, \quad N_2 = \frac{N + \Delta N}{2}$$

$$\frac{dN_1}{dt} = -\frac{dN_2}{dt}$$

$$\frac{d(N_2 - N_1)}{dt} = \frac{d\Delta N}{dt} = 2\left(W_p N_1 - \frac{N_2}{\tau_{21}}\right)$$

$$= 2\left[W_p\left(\frac{N - \Delta N}{2}\right) - \frac{(N + \Delta N)/2}{\tau_{21}}\right]$$

$$\frac{d\Delta N}{dt} = -\left(W_p + \frac{1}{\tau_{21}}\right)\Delta N + \left(W_p - \frac{1}{\tau_{21}}\right)N$$



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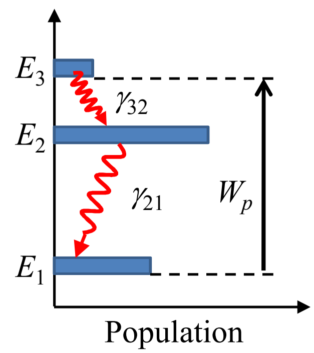
Steady-State

For constant pump intensity

$$\frac{d}{dt}\Delta N = 0 \Rightarrow -\left[W_p + \frac{1}{\tau}\right]\Delta N_{ss} + \left[W_p - \frac{1}{\tau}\right]N = 0$$

$$\Delta N_{ss} = \frac{W_p\tau - 1}{W_p\tau + 1}N$$

$$\frac{d}{dt}\Delta N = -\left(W_p + \frac{1}{\tau}\right)\times\left[\Delta N(t) - \Delta N_{ss}\right]$$



Here ΔN decay rate depends on the pump rate as well as the lifetime decay time = $(W_p + 1/\tau_{21})^{-1}$. This is due to the fact that the lower level is not emptied out.

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Inversion From a Pulse

Here we start with $\Delta N(0) = -N$

$$\frac{d\Delta N}{dt} = -\left(W_p + \frac{1}{\tau_{21}}\right)(\Delta N - \Delta N_{ss}),\ \Delta N_{ss} = \frac{\tau_{21}W_p - 1}{\tau_{21}W_p + 1}N$$

$$\Delta N = Ae^{-\left(W_p + \frac{1}{\tau_{21}}\right)t} + B \Rightarrow -N = \Delta N(0) = A + B$$

$$B = \Delta N_{ss} = \frac{\tau_{21}W_p - 1}{\tau_{21}W_p + 1}, A = -\frac{\tau_{21}W_p - 1}{\tau_{21}W_p + 1}N - N$$

$$\frac{\Delta N}{N} = \frac{\left(\tau_{21}W_p - 1 - \tau_{21}W_p - 1\right)e^{-\left(W_p + \frac{1}{\tau_{21}}\right)t}}{\tau_{21}W_p - 1} + \frac{\tau_{21}W_p - 1}{\tau_{21}W_p + 1}$$

$$= \frac{\tau_{21}W_p - 1 - 2\tau_{21}W_p e^{-\left(W_p + \frac{1}{\tau_{21}}\right)t}}{\tau_{21}W_p + 1}$$

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Limits

Here we assume that $W_p >> 1/\tau$, i.e., the pumping rate exceeds the lifetime and $T_p/\tau << 1$, i.e., the pump time is short compared to the lifetime.

$$\frac{\Delta N(T_p)}{N} = \frac{W_p\tau_{21} - 1 - 2W_p\tau_{21}\exp\left[-\left(W_p\tau + 1\right)\frac{T_p}{\tau}\right]}{W_p\tau_{21} + 1} \approx 1 - 2e^{-W_pT_p}$$

So the ΔN achieved depends on W_pT_p (proportional to total energy) and that if it is large enough total inversion can be achieved.

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LASER AMPLIFICATION

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Laser Amplification and Gain

- Now we have the means to build a laser amplifier.
- Strong (usually) electric dipole transitions.
- Pumping methods for achieving population inversion.
- Now we have a means of generating power per unit volume by stimulated emission.

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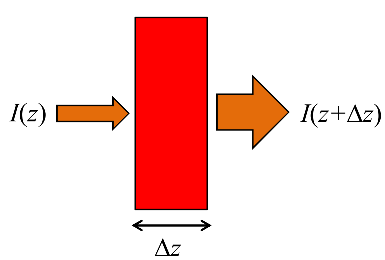
Gain, Absorption

Let us consider a plane wave going through a medium. We will see that power added to or absorbed from the field.



I : Intensity

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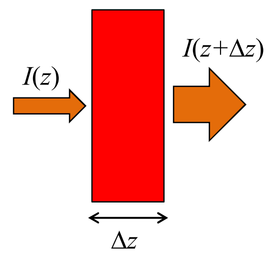
Rate of Change with Distance

Change in power

$$\Delta P = A\{I(z+\Delta z) - I(z)\} = A\Delta z 2gI$$

$$A\frac{dI}{dz}\Delta z = A\Delta z 2gI, \qquad 2g \propto K, n \propto I$$

$$\frac{dI}{dz} = 2gI \Rightarrow I = I_0 e^{2gz}$$



$2g$: Gain coefficient

We have seen that $2g$ is proportional to $(N_2 - N_1)$. Usually when we pass light through a medium it is absorbed since the atoms are in the ground state, i.e., $N_1 >> N_2$ or $2g < 0$. Note if we can invert the population by making $N_2 > N_1$ then $2g > 0$ and we get exponential gain rather than loss!!

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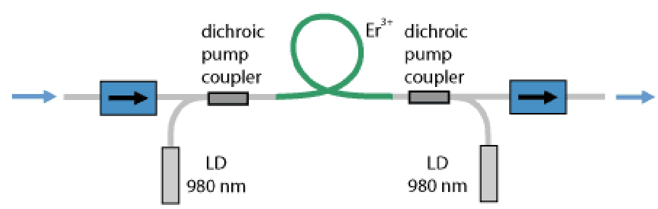
Laser Amplifiers



Thus if we have population inversion, we have power per unit volume generated and added in phase with the incoming field. This process allows us to amplify weak optical signals. In optical fibers, the amplifiers are typically made from Erbium ions doped into the fiber. External laser light (at a wavelength shorter than the signal's) is coupled into the fiber to optically pump the erbium ions. These amplifiers are spaced typically somewhere between 25 to 100 km apart. At the operational wavelength near 1550 nm modern optical fiber has a loss of 0.2 dB/km yielding about 20 dB of loss in 100 km. The optical amplifier then overcomes this loss to bring the signal back up to its initial level. Then the signal propagates another distance and is then re-amplified. Note that trans-oceanic optical communications work exactly this way with amplifiers spaced about 50 km apart sitting on the bottom of the ocean!! Commercially as many as 64 channels at 10 Gb/s can operate on a signal fiber on these links (much more is possible)!!

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Noise

Of course there is a problem introduced by repeated amplification. In addition to the stimulated emission there is spontaneous emission which is random. The fraction of the spontaneous emission that is in the direction (same mode as) of the signal will then show up as noise. We can reduce some of the noise by optically filtering it out but some remains at the same wavelength as the signal. Fortunately the extra noise added per amplifier is small and thus one can use many amplifiers in cascade. However, the noise does accumulate.

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Single Frequency Fields

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + j\omega \vec{D}$$

$$\vec{J} = \sigma \vec{E}$$

$$\begin{aligned}\vec{D}(\omega) &= \varepsilon_b(\omega)\vec{E}(\omega) + \vec{P}_a(\omega) \\ &= \varepsilon_b(\omega)\vec{E}(\omega) + \varepsilon_0 \chi_a(\omega)\vec{E}(\omega) \\ &= \varepsilon_b(\omega)\left(1 + \frac{\varepsilon_0}{\varepsilon_b(\omega)}\chi_a(\omega)\right)\vec{E}(\omega)\end{aligned}$$

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Review EM

Here we will use plane waves with propagation in one dimension, the z direction.

$$\nabla\times\nabla\times\vec{\mathrm{E}} = -\nabla\times\frac{\partial\vec{\mathrm{B}}}{\partial t} = -j\omega\nabla\times\vec{\mathrm{B}}$$

$$\nabla\times\vec{\mathrm{H}} = \vec{\mathrm{J}} + \frac{\partial\vec{\mathrm{D}}}{\partial t} = \vec{\mathrm{J}} + j\omega\vec{\mathrm{D}}$$

$$\nabla\times\nabla\times\vec{\mathrm{E}} = \nabla(\nabla\cdot\vec{\mathrm{E}}) - \nabla^2\vec{\mathrm{E}}$$

Now if the dielectric constant does not vary with distance then

$\nabla\cdot\vec{\mathrm{E}} = 0.$

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Wave Equation

$$\begin{aligned}-\nabla^2\vec{\mathrm{E}} &= -j\omega\nabla\times\vec{\mathrm{B}} = -j\omega\mu\nabla\times\vec{\mathrm{H}}\\ &= -j\omega\mu\left(\vec{\mathrm{J}} + j\omega\vec{\mathrm{D}}\right)\\ &= -j\omega\mu\left[\sigma\vec{\mathrm{E}} + j\omega\varepsilon_b\left(1+\frac{\varepsilon_0}{\varepsilon_b}\chi_a\right)\vec{\mathrm{E}}\right]\\ &= \omega^2\mu\varepsilon_b\left[1+\frac{\varepsilon_0}{\varepsilon_b}\chi_a - j\frac{\sigma}{\omega\varepsilon_b}\right]\vec{\mathrm{E}}\end{aligned}$$

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One Dimension

Using the plane wave approximation and assuming both the background material and the atoms leading to gain provide isotropic response we can reduce this to a simple one dimensional equation. Also we can then use linear polarized light without loss of generality.

$$-\frac{\partial^2 E}{\partial z^2} = \omega^2 \mu \varepsilon_b \left[1 + \frac{\varepsilon_0}{\varepsilon_b} \chi_a - j \frac{\sigma}{\omega \varepsilon_b} \right] E(z)$$

$$\frac{\partial^2 E}{\partial z^2} + \omega^2 \mu \varepsilon_b \left[1 + \frac{\varepsilon_0}{\varepsilon_b} \chi_a - j \frac{\sigma}{\omega \varepsilon_b} \right] E(z) = 0$$

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$\sigma = 0, \quad \chi = 0$

If we have no conductivity and no susceptibility from the atoms, i.e., just a lossless background dielectric

$$\frac{\partial^2 E}{\partial z^2} + \omega^2 \mu \varepsilon_b E(z) = 0$$

$$E = E_0 e^{\pm jkz} \Rightarrow -k^2 + \omega^2 \mu \varepsilon_b = 0$$

$$k = \sqrt{\mu \varepsilon_b} \omega = n \frac{\omega}{c}, n = \sqrt{\frac{\mu \varepsilon_b}{\mu_0 \varepsilon_0}}$$

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$\sigma \neq 0, \quad \chi \neq 0$

The conductivity and susceptibility will in general be small so we can approximate

$$\frac{\partial^2 E}{\partial z^2} + \omega^2 \mu \varepsilon_b \left[1 + \frac{\varepsilon_0}{\varepsilon_b} \chi - j \frac{\sigma}{\omega \varepsilon_b} \right] E(z) = 0$$

$$E = E_0 e^{\pm jkz} \Rightarrow k^2 = \omega^2 \mu \varepsilon_b \left[1 + \frac{\varepsilon_0}{\varepsilon_b} \chi - j \frac{\sigma}{\omega \varepsilon_b} \right]$$

$$\frac{\varepsilon_0}{\varepsilon_b} |\chi| \ll 1, \frac{|\sigma|}{\omega \varepsilon_b} \ll 1, k \approx n \frac{\omega}{c} \left[1 + \frac{\varepsilon_0}{2\varepsilon_b} \chi - j \frac{\sigma}{2\omega \varepsilon_b} \right], \chi = \chi' + j\chi''$$

$$k \approx n \frac{\omega}{c} \left[1 + \frac{\varepsilon_0}{2\varepsilon_b} (\chi' + j\chi'') - j \frac{\sigma}{2\omega \varepsilon_b} \right]$$

$$= n \frac{\omega}{c} \left(1 + \frac{\varepsilon_0}{2\varepsilon_b} \chi' \right) - jn \frac{\omega}{c} \left(\frac{\sigma}{2\omega \varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b} \chi'' \right)$$

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Loss

Now let's take propagation for simplicity in the +*z* direction so our propagating wave is of the form $e^{j(\omega t - kz)}$

$$k = n \frac{\omega}{c} \left(1 + \frac{\varepsilon_0}{2\varepsilon_b} \chi' \right) - jn \frac{\omega}{c} \left(\frac{\sigma}{2\omega \varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b} \chi'' \right)$$

$$e^{j(\omega t - kz)} = e^{j\left[\omega t - n\frac{\omega}{c}\left(1 + \frac{\varepsilon_0}{2\varepsilon_b}\chi'\right)z \right]} e^{-n\frac{\omega}{c}\left(\frac{\sigma}{2\omega\varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b}\chi''\right)z}$$

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Additional Phase

Now we see that the propagation gets an additional phase shift as a function of propagation distance z.

$$\delta\phi = n\frac{\omega}{c}\left(\frac{\varepsilon_0}{2\varepsilon_b}\chi'\right)z$$

$$\frac{\delta\phi}{\phi} = \frac{n\frac{\omega}{c}\left(\frac{\varepsilon_0}{2\varepsilon_b}\chi'\right)z}{n\frac{\omega}{c}z} = \frac{\varepsilon_0}{2\varepsilon_b}\chi'$$

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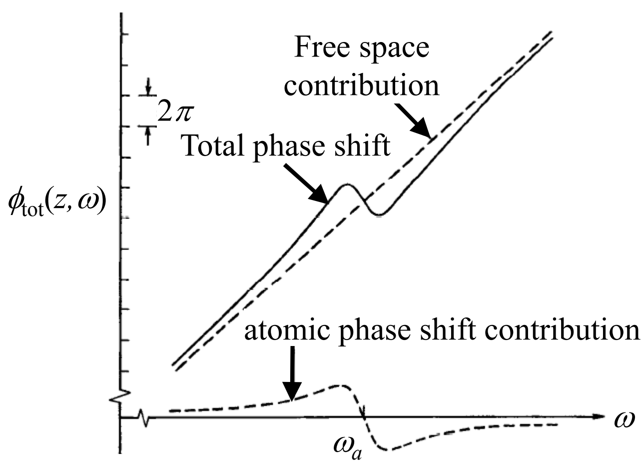
Change Near Resonance



$$\chi' = \frac{3}{4\pi^2}\frac{(N_2 - N_1)\lambda^3\gamma_{\text{rad}}}{\Delta\omega_a}\frac{2(\omega-\omega_a)/\Delta\omega_a}{1+\left[2(\omega-\omega_a)/\Delta\omega_a\right]^2}$$

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Gain or Loss

The term in the exponent with the conductivity represents loss. Assume that $\chi = 0$ then it is obvious that we have loss and the loss per unit length is $n\sigma/(2c\varepsilon)$ for the field and $n\sigma/(c\varepsilon)$ for the power.

$$E(z) = E_0 e^{-n\frac{\omega}{c}\left[\frac{\sigma}{2\omega\varepsilon_b} - \frac{\varepsilon_0}{2\varepsilon_b(\omega)}\chi''\right]z}$$

$$\sigma = 0 \Rightarrow E(z) = E_0 e^{n\frac{\omega}{c}\frac{\varepsilon_0\chi''}{2\varepsilon_b(\omega)}z}$$

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Gain

Suppose we have a medium with gain. Then let's look at the field and power gain after a certain length L of propagation.

$$\tilde{g}(\omega) = \frac{\tilde{E}(L)}{\tilde{E}(0)}$$

$$G(\omega) = \frac{I(L)}{I(0)} = |\tilde{g}(\omega)|^2 = e^{[2\alpha_m(\omega) - 2\alpha_0]L}$$

$$\alpha_m(\omega) = n\frac{\omega\varepsilon_0}{2c\varepsilon_b}\chi''(\omega) = n\frac{\omega\varepsilon_0}{2c\varepsilon_b}\frac{\chi_0''}{1 + \left[2\frac{\omega - \omega_a}{\Delta\omega_a}\right]^2}$$

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Gain Lineshape

$$G(\omega) = e^{\left[n \frac{\omega \varepsilon_0 L \chi_0''}{c \varepsilon_b} \frac{1}{1 + \left[2(\omega - \omega_a)/\Delta\omega_a \right]^2} \right]}$$

- Note that the lorentzian lineshape appears in the exponent.
- The exponential gain falls off much more rapidly with detuning than the atomic lineshape itself → gain narrowing.

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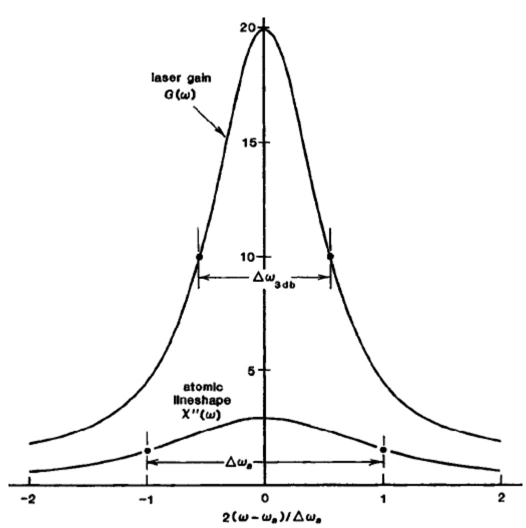
Gain Narrowing



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Bandwidth

$$G_{\mathrm{dB}}(\omega) = \frac{G_{\mathrm{dB}}(\omega_a)}{1+\left[2(\omega-\omega_a)/\Delta\omega_a\right]^2} = G_{\mathrm{dB}}(\omega_a)-3$$

$$(\omega-\omega_a)_{3\mathrm{dB}} = \pm\frac{\Delta\omega_a}{2}\sqrt{\frac{3}{G_{\mathrm{dB}}(\omega_a)-3}}$$

Full 3-dB bandwidth

$$\Delta\omega_{3\mathrm{dB}} = \Delta\omega_a\sqrt{\frac{3}{G_{\mathrm{dB}}(\omega_a)-3}}$$

Project

- Progress discussion: **2 September 2019**.
- Presentation and Report submission: **23 September 2019**