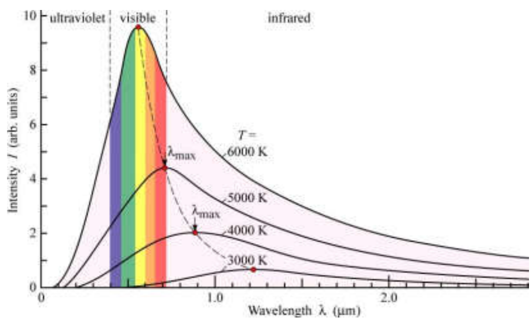
BLACKBODY RADIATION

Blackbody Radiation

- Every object at a finite temperature radiates light.
- Hotter objects emit more light and at smaller wavelengths when compared to cooler objects.



Blackbody Radiation Density

The number of modes/unit frequency/volume $= \frac{8\pi\nu^2}{c^3}$

Average energy at frequency ν is $\frac{h\nu}{\exp(h\nu/kT)-1}$

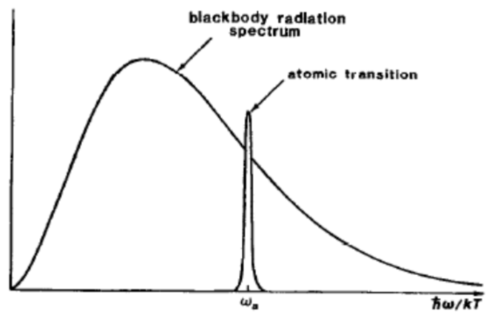
Radiation density $dU_{\text{bbr}} = \frac{8\pi}{\lambda^3}\frac{\hbar\, d\omega}{\exp\left(\hbar\omega/kT_{\text{rad}}\right)-1}$

$$dU_{\text{bbr}} = \frac{\varepsilon}{2} d\,|\tilde{E}_{\text{bbr}}|^2$$

$$d\,|\tilde{E}_{\text{bbr}}|^2 = \frac{16\pi}{\lambda^3}\frac{\varepsilon\hbar\, d\omega}{\exp\left(\hbar\omega/kT_{\text{rad}}\right)-1}$$

Blackbody-Stimulated Transitions

Any atoms that may be present in the region under consideration are then exposed to these entirely real though noise-like E_{bbr} fields.



Blackbody-Stimulated Transitions

Any atoms that may be present in the region under consideration are then exposed to these entirely real though noise-like E_{bbr} fields.

$$dW_{12,\text{bbr}} = dW_{21,\text{bbr}} = \frac{3^*}{8\pi^2}\frac{\gamma_{\text{rad}}}{\hbar\Delta\omega_a}\frac{\varepsilon d\,|\tilde{E}_{\text{bbr}}|^2\,\lambda^3}{1+\left[2(\omega-\omega_a)/\Delta\omega_a\right]^2}$$

Here 3* = 1 since E fields are randomly polarized.

$$W_{12,\text{bbr}} = W_{21,\text{bbr}} = \frac{\gamma_{\text{rad}}}{\exp\left(\hbar\omega_a/kT_{\text{rad}}\right)-1}\int_{-\infty}^{\infty}\frac{2}{\pi\Delta\omega_a}\frac{d\omega}{1+\left[2(\omega-\omega_a)/\Delta\omega_a\right]^2}$$

$$W_{12,\text{bbr}} = W_{21,\text{bbr}} = \frac{\gamma_{\text{rad}}}{\exp\left(\hbar\omega_a/kT_{\text{rad}}\right)-1}$$

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Power Absorption

$$W_{12,\text{bbr}} = W_{21,\text{bbr}} = \frac{\gamma_{\text{rad}}}{\exp\left(\hbar\omega_a/kT_{\text{rad}}\right)-1}$$

- It seems that there will be net power absorption, proportional to the atomic population difference $\Delta N = N_1 - N_2$.
- Blackbody field will be continuously delivering energy.
- Equilibrium?

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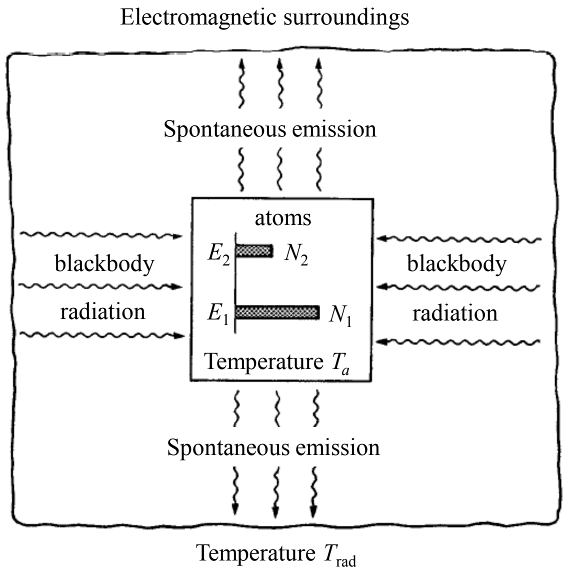
Power Emission

- The equilibrium is preserved since there are spontaneous or downward transition rate γ_{rad}.
- Power is transferred back to the electromagnetic surroundings.

Thermal Balance



Thermal Balance

$$\frac{\text{energy flow out of atoms}}{\text{energy flow into atoms}} = \frac{\left(W_{21,\text{bbr}} + \gamma_{\text{rad}}\right) N_2}{W_{12,\text{bbr}} N_1}$$

Boltzmann ratio → $$\frac{N_2}{N_1} = \exp\left(-\frac{\hbar\omega_a}{kT_a}\right)$$

Blackbody → $$\frac{W_{21,\text{bbr}} + \gamma_{\text{rad}}}{W_{12,\text{bbr}}} = \exp\left(\frac{\hbar\omega_a}{kT_{\text{rad}}}\right)$$

$$\frac{\text{energy flow out of atoms}}{\text{energy flow into atoms}} = \exp\left(\frac{\hbar\omega_a}{kT_{\text{rad}}} - \frac{\hbar\omega_a}{kT_a}\right)$$

Rates are equal when $T_a = T_{\text{rad}}$

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RELAXATION PROCESSES

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Upward Relaxation

- So far, we have considered only the downward relaxations.
- In a complete description, when an atom is coupled to external surroundings it can do more than just relax downward and give energy to those surroundings. It can also (but with inherently lower probability) receive energy from its thermal surroundings and be lifted or relaxed upward in energy. This is directly related to the fact in thermal equilibrium there are always some number of atoms, given by Boltzmann ratios, in upper energy levels.
- For optical-frequency transitions at room temperature, Boltzmann ratio is enormously small ($\approx 10^{-36}$). Only downward transition rates may be considered.

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Nonradiative Relaxation

- **Stimulated:** Phonon coupling

$$W_{12,\mathrm{nr}} = W_{21,\mathrm{nr}} = \frac{\gamma_{\mathrm{nr}}}{\exp\left(\hbar\omega_a / kT_{\mathrm{nr}}\right) - 1}$$

- **Spontaneous:** Inelastic collisions in gases, electron-electron scattering in semiconductor lasers, scattering due to interface roughness.

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Total Relaxation Rates

- Transition rate in the downward direction

$$\left.\frac{dN_2}{dt}\right|_{\substack{\text{downward}\\ \text{relaxation}}} = \left(W_{21,\text{bbr}} + \gamma_{\text{rad}} + W_{21,\text{nr}} + \gamma_{\text{nr}}\right) N_2 = w_{21} N_2$$

- Corresponding flow rate in the upward direction

$$\left.\frac{dN_1}{dt}\right|_{\substack{\text{upward}\\ \text{relaxation}}} = \left(W_{12,\text{bbr}} + W_{12,\text{nr}}\right) N_1 = w_{12} N_1$$

- In general $w_{ij} = W_{ij,\text{bbr}} + W_{ij,\text{nr}}$

$$w_{ji} = W_{ji,\text{bbr}} + \gamma_{ji,\text{rad}} + W_{ji,\text{nr}} + \gamma_{ji,\text{nr}}$$

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Notations

w_{ij} :

- Total relaxation transition probabilities (per atom and per unit time) in the upward and downward directions due to purely thermal interactions plus energy decay processes connecting the atoms to their surroundings.
- We cannot turn off w_{ij}'s (except possibly cooling the surroundings).

W_{ij} :

- Signal-stimulated transition probabilities that are produced by external signals or pumping mechanisms.
- We can turn off W_{ij}'s.

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Boltzmann Relaxation Ratio

$$\frac{w_{12}(\uparrow)}{w_{21}(\downarrow)} = \frac{W_{12,\text{bbr}} + W_{12,\text{nr}}}{W_{21,\text{bbr}} + W_{21,\text{nr}} + \gamma_{\text{rad}} + \gamma_{\text{nr}}}$$

$$= \frac{\dfrac{\gamma_{\text{rad}}}{\exp(\hbar\omega_a / kT_{\text{rad}}) - 1} + \dfrac{\gamma_{\text{nr}}}{\exp(\hbar\omega_a / kT_{\text{nr}}) - 1}}{\dfrac{\gamma_{\text{rad}}}{\exp(\hbar\omega_a / kT_{\text{rad}}) - 1} + \dfrac{\gamma_{\text{nr}}}{\exp(\hbar\omega_a / kT_{\text{nr}}) - 1} + \gamma_{\text{rad}} + \gamma_{\text{nr}}}$$

- If $T_{\text{nr}} = T_{\text{rad}} = T$

$$\frac{w_{12}(\uparrow)}{w_{21}(\downarrow)} = e^{-\hbar\omega_a / kT}$$

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Optical Frequency

- For visible frequencies

$$\frac{\hbar\omega_a}{k} \approx 25000$$

- At room temperature

$$\frac{w_{12}(\uparrow)}{w_{21}(\downarrow)} = e^{-\hbar\omega_a / kT} \approx e^{-\frac{25000}{300}} \approx 10^{-36}$$

$$w_{12}(\uparrow) \approx 0$$

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TWO-LEVEL SYSTEM

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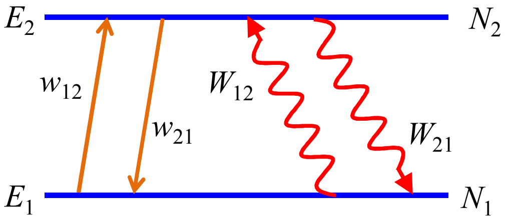
Rate Equation



$$\frac{dN_1(t)}{dt} = -\left[W_{12} + w_{12}\right]N_1(t) + \left[W_{21} + w_{21}\right]N_2(t)$$

$$\frac{dN_2(t)}{dt} = \left[W_{12} + w_{12}\right]N_1(t) - \left[W_{21} + w_{21}\right]N_2(t)$$

$$N_1(t) + N_2(t) = N, \qquad N_1(t) - N_2(t) = \Delta N$$

$$N_1(t) = \frac{N + \Delta N}{2}, \qquad N_2(t) = \frac{N - \Delta N}{2}$$

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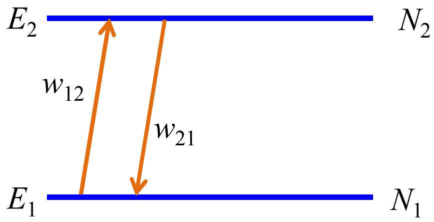
No External Signal

$W_{12} = W_{21} = 0$

At steady-state

$$\frac{dN_1}{dt} = 0$$



$$w_{12}N_{10} = w_{21}N_{20},\ \ \Delta N_0 = N_{10} - N_{20} = N_{10} - \frac{w_{12}}{w_{21}}N_{10} = N_{10}\left(1 - \frac{w_{12}}{w_{21}}\right)$$

$$N = N_{10} + N_{20} = N_{10}\left(1 + \frac{w_{12}}{w_{21}}\right) \Rightarrow \Delta N_0 = N\left(1 - \frac{w_{12}}{w_{21}}\right) / \left(1 + \frac{w_{12}}{w_{21}}\right) = \frac{w_{21} - w_{12}}{w_{21} + w_{12}}N$$

$$\Delta N_0 = \frac{1 - e^{-\hbar\omega_a/kT}}{1 + e^{-\hbar\omega_a/kT}}N = \frac{e^{\hbar\omega_a/2kT} - e^{-\hbar\omega_a/2kT}}{e^{\hbar\omega_a/2kT} + e^{-\hbar\omega_a/2kT}}N = N\tanh\left(\frac{\hbar\omega_a}{2kT}\right)$$

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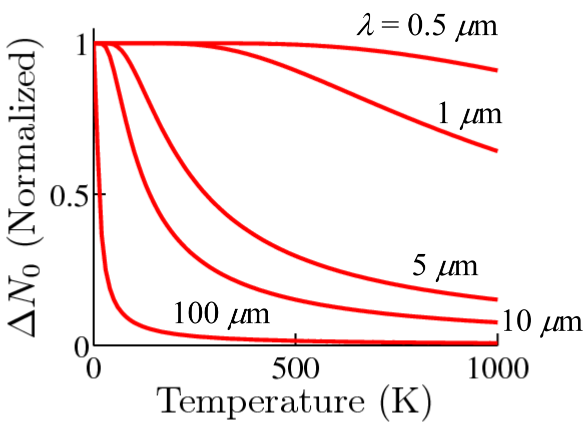
ΔN_0

$$\Delta N_0 = N\tanh\left(\frac{\hbar\omega_a}{2kT}\right)$$



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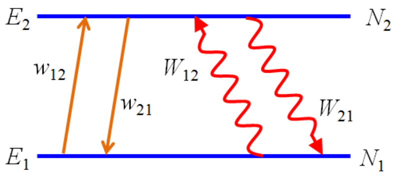
With External Signal



$$N_1 = \frac{N + \Delta N}{2}$$

$$\frac{dN_1}{dt} = \frac{1}{2}\frac{d\Delta N}{dt}$$

$$\frac{1}{2}\frac{d\Delta N}{dt} = -\left[W_{12} + w_{12}\right]\left[\frac{N + \Delta N}{2}\right] + \left[W_{21} + w_{21}\right]\left[\frac{N - \Delta N}{2}\right]$$

$$\frac{1}{2}\frac{d\Delta N}{dt} = -\left[W_{12} + W_{21}\right]\frac{\Delta N}{2} - \left(w_{12} + w_{21}\right)\frac{\Delta N}{2} + \left(w_{21} - w_{12}\right)\frac{N}{2}$$

$$\frac{d\Delta N}{dt} = -2W_{12}\Delta N - \left(w_{12} + w_{21}\right)\left[\Delta N - \frac{w_{21} - w_{12}}{w_{21} + w_{12}} N\right]$$

$$\frac{d\Delta N}{dt} = -2W_{12}\Delta N - \frac{\Delta N - \Delta N_0}{T_1}$$

$$w_{12} + w_{21} = \frac{1}{T_1}$$

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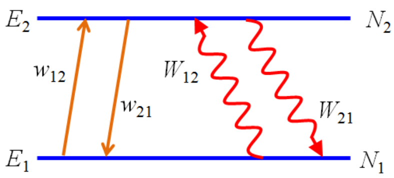
T_1

$$\frac{d\Delta N}{dt} = -2W_{12}\Delta N - \frac{\Delta N - \Delta N_0}{T_1}$$



- With no applied signal, $\Delta N(t)$ relaxes to ΔN_0 with an exponential time constant T_1.
- T_1 is often called the population recovery time.
- For an optical frequency transition

$$\frac{1}{T_1} = w_{12} + w_{21} \approx \gamma_{21} = \frac{1}{\tau_{21}}$$

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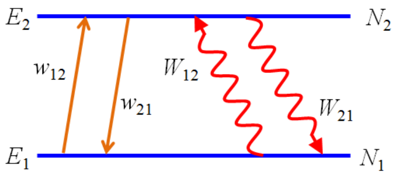
$2W_{12}$

$$\frac{d\Delta N}{dt} = -2W_{12}\Delta N - \frac{\Delta N - \Delta N_0}{T_1}$$



- Drive the population difference $\Delta N(t)$ to zero, i.e., to saturate the population difference.
- Note that the factor 2 appears in front of this stimulated term because the transition of a single atom from level 1 to level 2 both reduces $N_1(t)$ by one and increases $N_2(t)$ by one, and thus changes $\Delta N(t)$ by twice that much.

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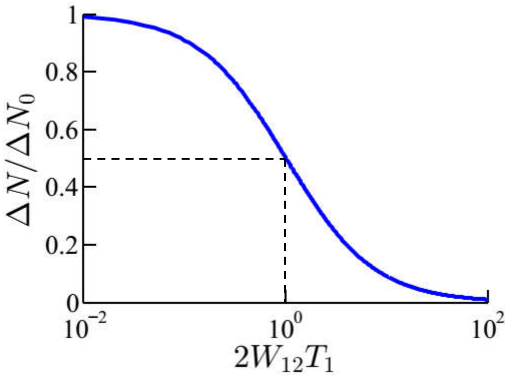
Steady-State

$$\frac{d}{dt}\Delta N(t) = 0 = -2W_{12}\Delta N(t) - \frac{\Delta N(t) - \Delta N_0}{T_1}$$

$$\Delta N = \Delta N_{ss} = \Delta N_0 \frac{1}{1 + 2W_{12}T_1}$$

- ΔN is driven toward zero at large enough applied signal.



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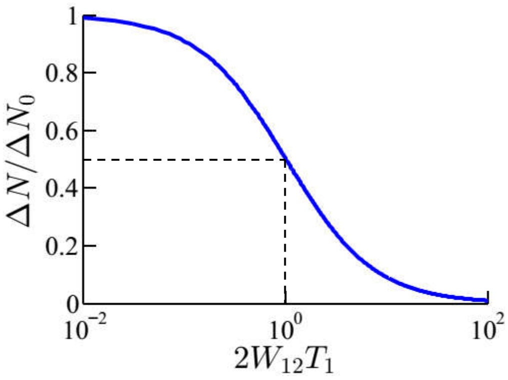
Saturation

$$\frac{\Delta N_{ss}}{\Delta N_0} = \frac{1}{1+2W_{12}T_1} = \frac{1}{1+\dfrac{W_{12}}{W_{sat}}} = \frac{1}{1+\text{const}\times\text{signal power}}$$

- $W_{sat} = 1/2T_1$ is the value of the stimulated-transition probability at which the population difference is driven down to exactly half its initial or small-signal value.



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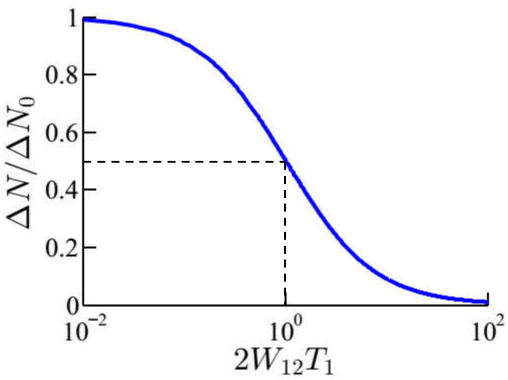
Saturation

$$\alpha_m = \alpha_{m0}\frac{1}{1+\dfrac{I}{I_{sat}}} = \alpha_{m0}\frac{1}{1+\text{const}\times\text{signal power}}$$

- I_{sat}: Saturation intensity at which the gain or loss coefficient is saturated down to half of its initial value.



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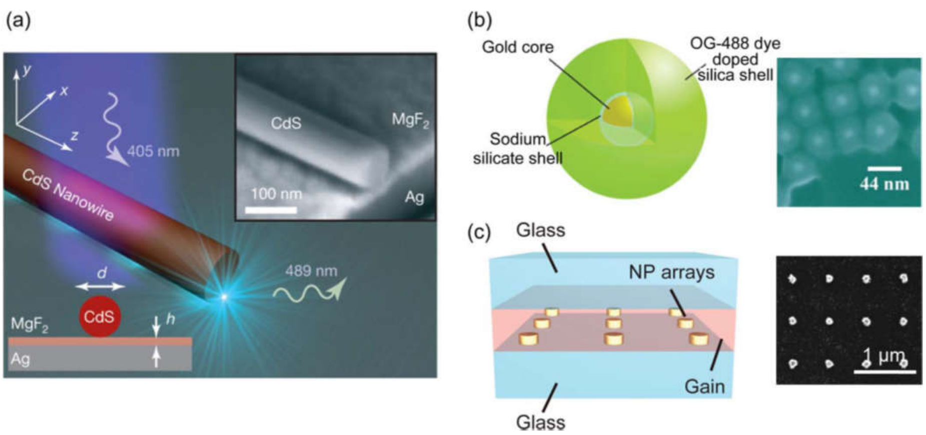
PROJECT

Topic: Nanolasers



Project

- Pick a topic on nanolasers.
- Group size: two.
- Proposal submission: 29 July 2019.
- Final submission: TBD.

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