LASER OSCILLATION

Threshold Conditions

The basic requirement either for just reaching laser oscillation threshold, or for maintaining steady-state laser oscillation, is that the round-trip gain inside the laser cavity, including mirror reflections, must be exactly unity, modulo an integer number of multiples of $e^{-j2\pi}$.

$$\tilde{g}_{rt}(\omega) = r_1 r_2 e^{\left[\alpha_m(\omega)p_m - \alpha_0 p - j\frac{\omega p}{c} - j\Delta\beta_m(\omega)p_m\right]}$$
$$= e^{-jq2\pi}$$

Threshold Conditions

Amplitude condition:

$$r_1 r_2 e^{[\alpha_m(\omega) p_m - \alpha_0 p]} = 1$$

→ Determines the population inversion density, and hence the pumping rate, needed to reach oscillation threshold.

Phase condition:

$$j\frac{\omega p}{c} + j\Delta\beta_m(\omega) p_m = q2\pi$$

→ Determines primarily the frequency ω at which the laser must oscillate.

Inversion Density

From the amplitude condition:

$$2\alpha_m(\omega) p_m = 2\alpha_0 p + \ln\left[\frac{1}{R_1 R_2}\right]$$

$$2\alpha_m(\omega) p_m = \delta_m(\omega) = \delta_0 + \delta_1 + \delta_2 = \delta_c$$

Gain:

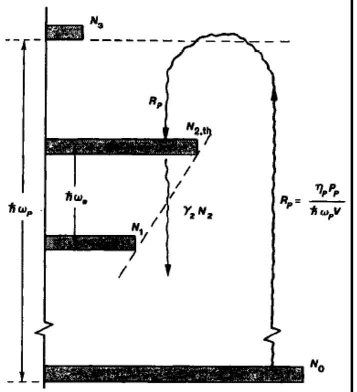
$$\alpha_m(\omega) = \frac{3^*}{4\pi^2} \frac{\Delta N \lambda^3 \gamma_{\text{rad}}}{\Delta\omega_a} \frac{1}{1 + \left[2\left(\omega - \omega_a\right) / \Delta\omega_a\right]^2}$$

Inversion Density

$$\Delta N = \Delta N_{\text{th}} = \frac{2\pi}{3} \times \frac{\Delta\omega_a}{\gamma_{\text{rad}}} \times \frac{1}{\lambda^3} \times \frac{\delta_c}{p_m}$$

For ΔN_{th} to be minimum:

- A narrow atomic linewidth $\Delta\omega_a$.
- A strong radiative decay rate γ_{rad}.
- A long wavelength λ.
- Low cavity losses and output coupling δ_c.
- A long gain medium p_m.



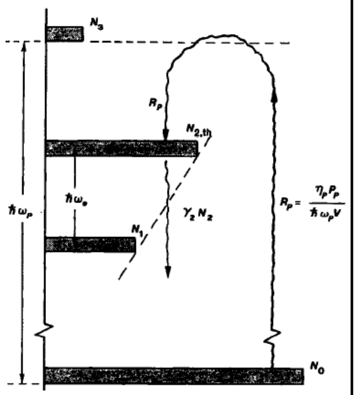
Threshold Pump Power Density

The effective pumping rate R_p and the upper level population density N_2 which is created by this pumping rate are given by

$$N_2 = \frac{R_p}{\gamma_2} = \frac{\eta_p P_p}{\gamma_2 \hbar\omega_p V}$$

Threshold pump power density

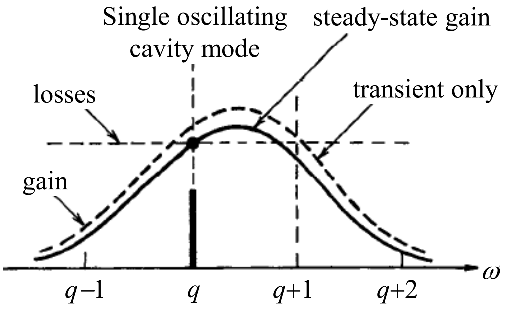
$$\frac{P_{p,\text{th}}}{V} = \frac{1}{\eta_p} \times \frac{N_{2,\text{th}}}{\Delta N_{\text{th}}} \times \frac{\omega_p}{\omega_a} \times \frac{\gamma_2}{\gamma_{\text{rad}}} \times \frac{4\pi^2}{3^*} \times \frac{\hbar\Delta\omega_a}{\lambda^3} \times \frac{c\delta_c}{p_m}$$



Oscillation Frequency

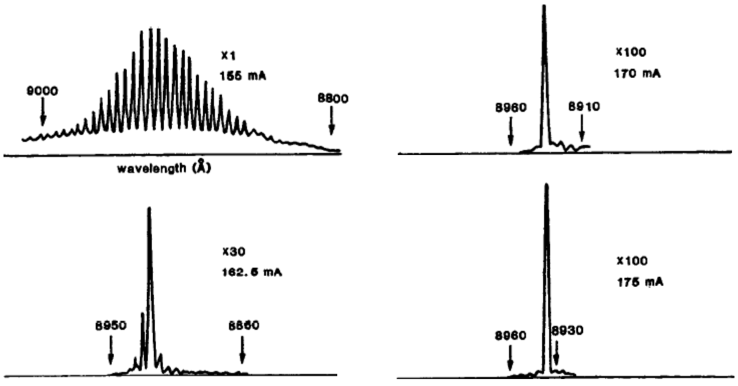
- Whether a laser will oscillate only in a single mode and at a single frequency, or in many modes at once.
- What are the exact frequencies of these oscillations will be atomic pulling effects are included.
- The answers depend whether the laser transition is homogeneously or inhomogeneously broadened.

Single Frequency Oscillation



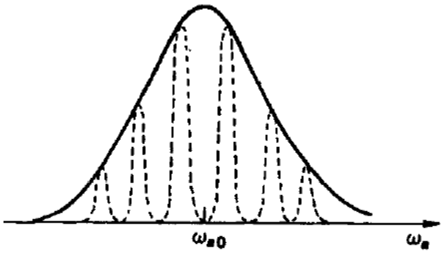
An ideally homogeneous laser oscillates under steady-state conditions in only one preferred mode, the first mode to reach the threshold. Pumping harder will make that preferred mode oscillate more strongly.

Single Frequency Oscillation



Single-mode oscillation rises up out of multimode amplified noise as the excitation current is increased.

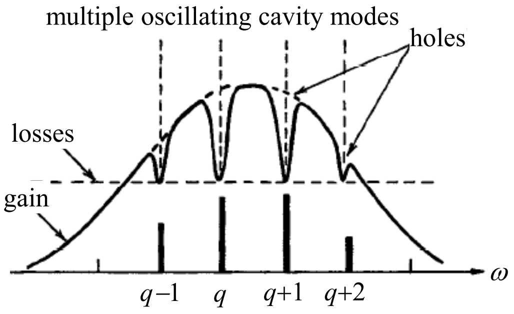
Inhomogeneous Broadening



Individual atomic responses, or “spectral packets,” within an inhomogeneously broadened atomic transition.

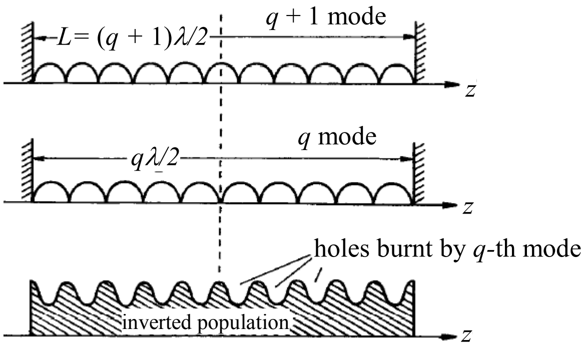
Multi-Mode Oscillation

Lasers with strongly inhomogeneous transitions can oscillate simultaneously on multiple frequencies or multiple axial modes.



Spatial Hole Burning

Spatial hole burning may lead to multimode operation even in homogeneous lasers.



Frequency Pulling

The round-trip phase shift $\phi(\omega)$ in a laser cavity with gain

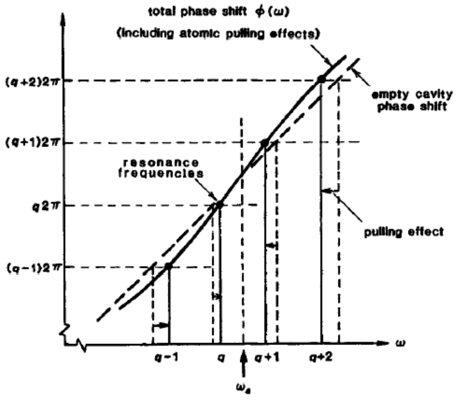
$$\phi(\omega) = \frac{\omega p}{c} + \Delta\beta_m(\omega)p_m = q2\pi$$

$$\Delta\beta_m(\omega) = \frac{\beta\chi'(\omega)}{2} = \frac{\omega\chi'(\omega)}{2c}$$

$$\omega = \omega_q' = \frac{q2\pi c/p}{1+\left(p_m/2p\right)\chi'\left(\omega_q'\right)}$$

$$\approx \frac{q2\pi c}{p} \times \left[1 - \frac{p_m}{p}\frac{\chi'\left(\omega_q'\right)}{2}\right]$$

$$= \omega_q + \delta\omega_q$$



Project

- Presentation: 7 Minutes (Max.)
- Report: 6 Pages (Max.)