GAUSS'S LAW

Gauss's Law – Application

Provides an easy means of finding $\vec{E}$ or $\vec{D}$ for symmetrical charge distribution.

A continuous charge distribution may have the following symmetry:

1. **Rectangular symmetry** $\rightarrow$ it depends only on x (or y or z)
2. **Cylindrical symmetry** $\rightarrow$ it depends only on ρ (independent of ϕ and z)
3. **Spherical symmetry** $\rightarrow$ it depends only on r (independent of θ and ϕ).

Whether the charge distribution is symmetric or not, Gauss's law always holds!

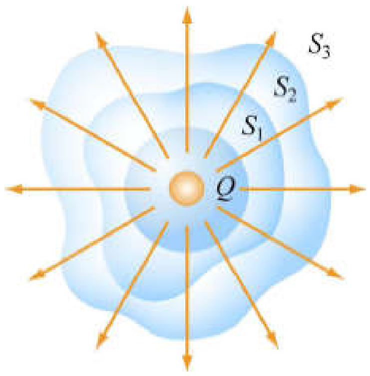
Gauss's Law – Application

1. Identify whether symmetry exists.
2. Choose Gaussian surface *S*.
3. Calculate
   - Charge enclosed by surface *S*
   - Apply Gauss's Law

$$Q = \oint_S \vec{D} \cdot d\vec{S}$$

3

Choosing Gaussian Surface



$$Q = \oint_S \vec{D} \cdot d\vec{S}$$

True for ALL surfaces.
Useful (to calculate $\vec{D}$) for SOME surfaces.

$\vec{D}$ is normal to the surface.

$$\vec{D} \cdot d\vec{S} = D\,dS \text{ or } -D\,dS$$

$\vec{D}$ is tangential to the surface.

$$\vec{D} \cdot d\vec{S} = 0$$

4

Symmetry & Gaussian Surfaces

Source Symmetry	Gaussian Surface
Spherical	Concentric Sphere
Cylindrical	Coaxial Cylinder
Planar	Gaussian "Pillbox"

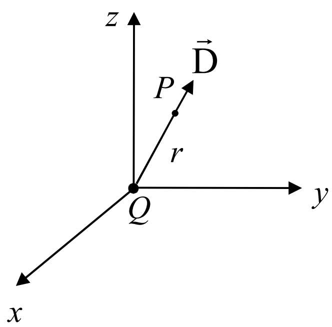
Point Charge

Identify symmetry → spherical

Choose Gaussian surface
A spherical surface centered at the origin

$\vec{D}$ is everywhere normal to the Gaussian surface

$$\vec{D} = D_r \hat{a}_r$$



Point Charge

Identify symmetry → spherical

Choose Gaussian surface
A spherical surface centered at the origin

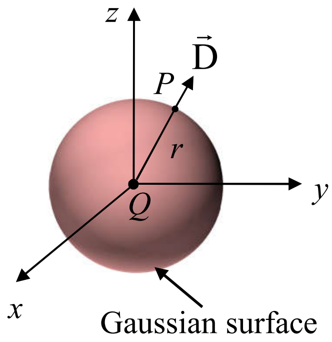
$\vec{D}$ is everywhere normal to the Gaussian surface

$$\vec{D} = D_r \hat{a}_r$$

Apply Gauss's Law

$$Q = \oint \vec{D} \cdot d\vec{S} = D_r \oint dS = D_r 4\pi r^2$$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2} \hat{a}_r}$$



7

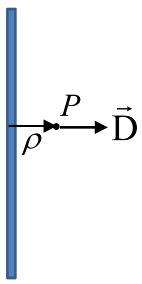
Infinite Line Charge

Identify symmetry → **cylindrical**

Choose Gaussian surface
A cylindrical surface with axis along the line

$\vec{D}$ is everywhere constant and normal to the Gaussian surface

$$\vec{D} = D_\rho \hat{a}_\rho$$



8

Infinite Line Charge

Identify symmetry → cylindrical

Choose Gaussian surface
A cylindrical surface with axis along the line

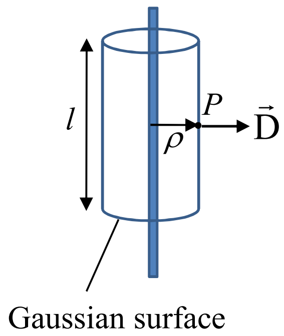
$\vec{D}$ is everywhere constant and normal to the Gaussian surface

$$\vec{D} = D_\rho \hat{a}_\rho$$

Apply Gauss's Law

$$\rho_L l = Q = \oint \vec{D} \cdot d\vec{S} = D_\rho \oint dS = D_\rho 2\pi\rho l$$

$$\boxed{\vec{D} = \frac{\rho_L}{2\pi\rho} \hat{a}_\rho}$$



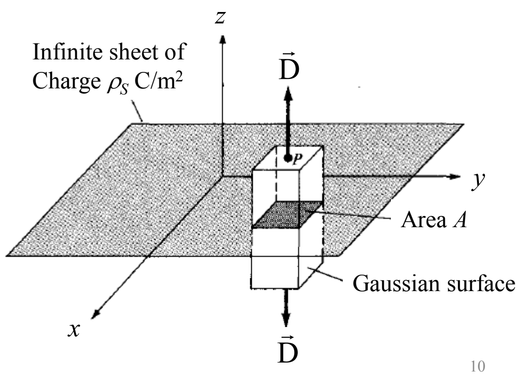
Infinite Sheet of Charge

Identify symmetry → planar

Choose Gaussian surface
A rectangular box or Gaussian "Pillbox"

$\vec{D}$ is everywhere constant and normal to the Gaussian surface

$$\vec{D} = D_z \hat{a}_z$$



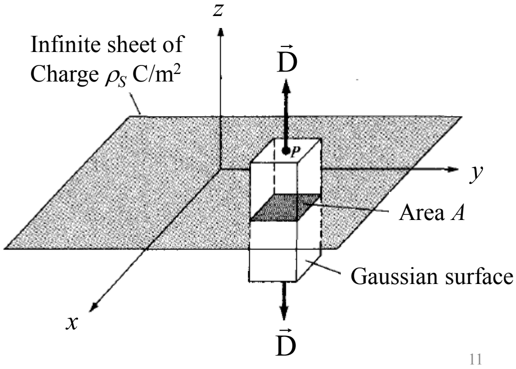
Infinite Sheet of Charge

Apply Gauss's Law

$$\rho_S \int dS = Q = \oint \vec{D} \cdot d\vec{S} = D_z \left[\int_{top} dS + \int_{bottom} dS \right]$$

$$\rho_S A = D_z (A + A)$$

$$\vec{D} = \frac{\rho_S}{2} \hat{a}_z$$



11

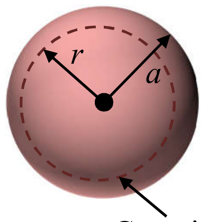
Uniformly Charged Sphere

Identify symmetry → spherical

Choose Gaussian surface
A spherical surface centered at the origin

$\vec{D}$ is everywhere normal to the Gaussian surface

$$\vec{D} = D_r \hat{a}_r$$

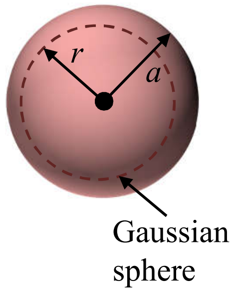


12

Uniformly Charged Sphere

Region 1: $r \leq a$

Draw Gaussian sphere in region 1 ($r \leq a$)



Note: r is arbitrary **but** is the radius for which you will calculate the $\vec{\mathrm{D}}$!

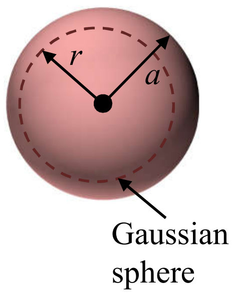
13

Uniformly Charged Sphere

Region 1: $r \leq a$

$$Q = \int \rho_v dv = \rho_v \int dv = \rho_v \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{r} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \rho_v \frac{4}{3} \pi r^3$$



$$\psi = \oint \vec{\mathrm{D}} \cdot d\vec{\mathrm{S}} = D_r \oint dS = D_r \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi$$

$$= D_r 4\pi r^2$$

Apply Gauss's Law

$$D_r 4\pi r^2 = \frac{4\pi r^3}{3} \rho_v$$

$$\vec{\mathrm{D}} = \frac{r}{3} \rho_v \, \hat{\mathrm{a}}_r, \quad 0 < r \leq a$$

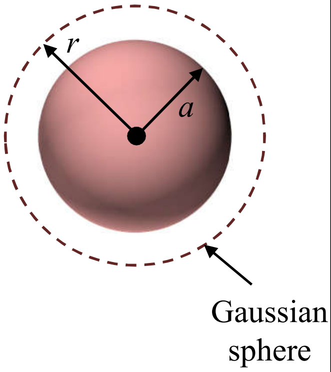
14

Uniformly Charged Sphere

Region 2: $r > a$

Draw Gaussian sphere in region 1 ($r > a$)

Again: Remember that r is arbitrary but is the radius for which you will calculate $\vec{D}$!



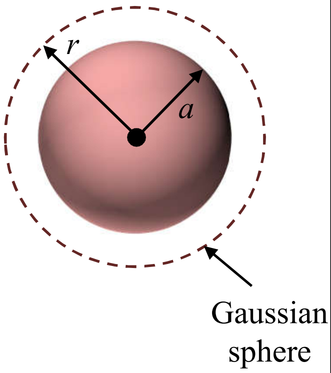
Uniformly Charged Sphere

Region 2: $r > a$

$$Q = \int \rho_v dv = \rho_v \int dv = \rho_v \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \rho_v \frac{4}{3} \pi a^3$$

$$\psi = \oint \vec{D} \cdot d\vec{S} = D_r 4\pi r^2$$

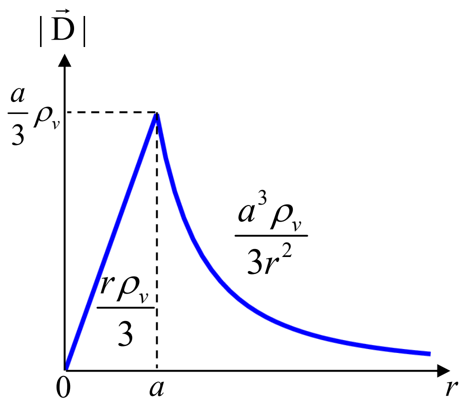


Apply Gauss's Law

$$D_r 4\pi r^2 = \frac{4\pi a^3}{3} \rho_v$$

$$\vec{D} = \frac{a^3}{3r^2} \rho_v \, \hat{a}_r, \quad r \geq a$$

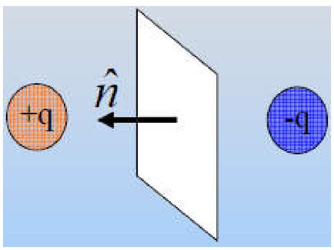
Uniformly Charged Sphere



17

Concept Question: Flux

The electric flux through the planar surface below (positive unit normal to left) is

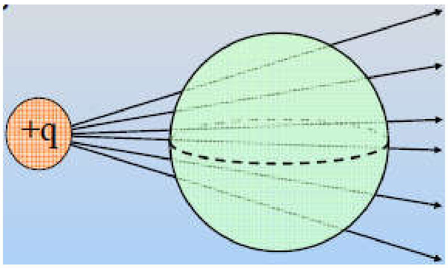


1. Positive
2. Negative
3. Zero
4. None of the above.

18

Concept Question: Flux thru Sphere

The total flux through the spherical surface is



1. Positive (net outward flux)
2. Negative (net inward flux)
3. Zero
4. None of the above.

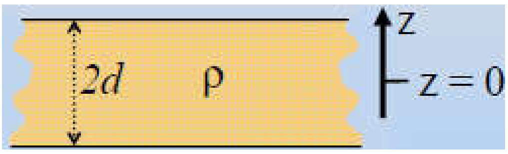
Concept Question: Charge in Pyramid

A pyramid has a square base of side a, and four faces which are equilateral triangles. A charge Q is placed on the center of the base of the pyramid. What is the net electric flux emerging from one of the triangular faces of the pyramid?

1. 0
2. $Q/8$
3. $Qa^2/2$
4. $Q/2$
5. Undetermined: we must know whether Q is infinitesimally above or below the plane?

Concept Question: Slab of Charge

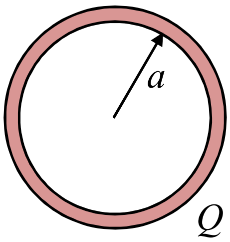
Consider positive, semi-infinite (in *x* and *y*) flat slab *z*-axis is perpendicular to the sheet, with center at $z = 0$. At the plane's center ($z = 0$), *E* field



1. Points in the positive *z*-direction
2. Points in the negative *z*-direction
3. Points in some other (x, y) direction
4. Is zero
5. Cannot be determined.

Concept Question: Spherical Shell

We just saw that in a solid sphere of charge the electric field grows linearly with distance. Inside the charged spherical shell at right ($r < a$) what does the electric field do?



1. Constant and Zero
2. Constant but Non-Zero
3. Still grows linearly
4. Some other functional form (use Gauss's Law)
5. Can't determine with Gauss's Law