CONTINUOUS CHARGE DISTRIBUTION

Ring of Charge

Coulomb's law:

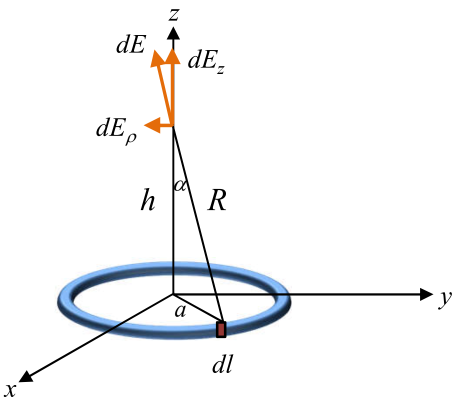
$$\vec{E} = \int \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2} \hat{a}_R$$

Distance and direction:

$$\vec{R} = a\left(-\hat{a}_\rho\right) + h\,\hat{a}_z$$

$$dl = a \, d\phi$$

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{\left(-a\,\hat{a}_\rho + h\,\hat{a}_z\right)}{\left[a^2 + h^2\right]^{3/2}} a \, d\phi$$



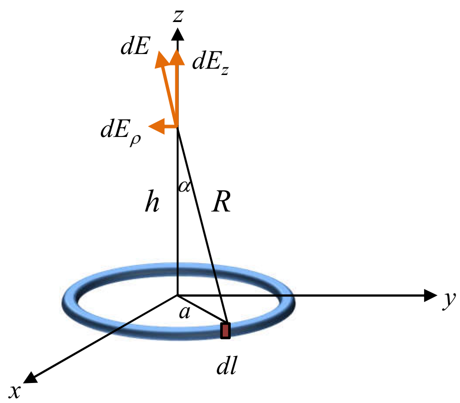
Ring of Charge

Symmetry:

$\hat{a}_\rho$ components add up to zero.

$$\vec{E} = \hat{a}_z \frac{\rho_L a h}{4\pi\varepsilon_0 \left[a^2 + h^2\right]^{3/2}} \int_0^{2\pi} d\phi$$

$$= \hat{a}_z \frac{\rho_L a h}{2\varepsilon_0 \left[a^2 + h^2\right]^{3/2}}$$



Total charge:

$$Q = \oint_L \rho_L \, dl = 2\pi a \rho_L$$

$$\rho_L = \frac{Q}{2\pi a}$$

$$\vec{E} = \hat{a}_z \frac{Q h}{4\pi\varepsilon_0 \left[a^2 + h^2\right]^{3/2}}$$

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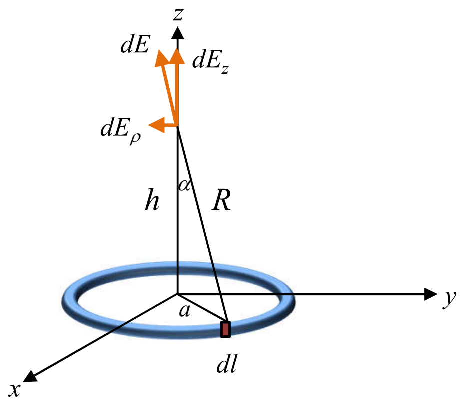
Ring of Charge

Special case:

As $a \rightarrow 0$

$$\vec{E} = \hat{a}_z \frac{Q}{4\pi\varepsilon_0 h^2} = \hat{a}_r \frac{Q}{4\pi\varepsilon_0 r^2}$$



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Surface Charge

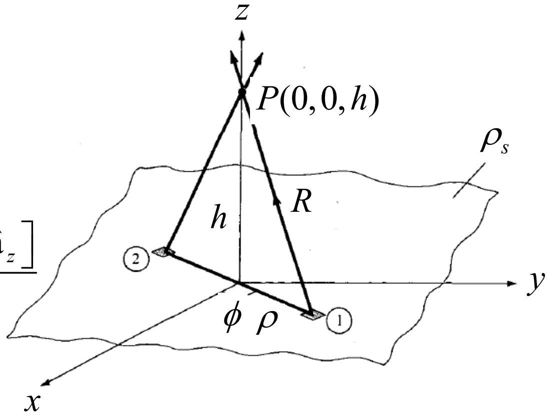
From Coulomb's law:

$$d\vec{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

Distance and Direction:

$$\vec{R} = \rho\left(-\hat{a}_\rho\right) + h\,\hat{a}_z$$

$$d\vec{E} = \frac{\rho_s \rho\, d\phi\, d\rho\left[-\rho\,\hat{a}_\rho + h\,\hat{a}_z\right]}{4\pi\varepsilon_0\left[\rho^2 + h^2\right]^{3/2}}$$



Symmetry:

$\hat{a}_\rho$ cancels for elements 1 and 2.

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Surface Charge

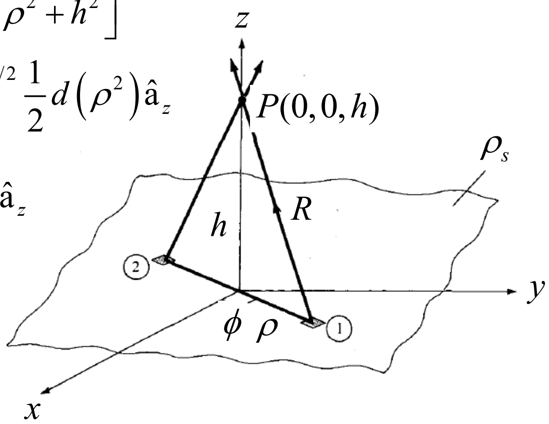
Integrate:

$$\vec{E} = \int d\vec{E}_z = \frac{\rho_s}{4\pi\varepsilon_0}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\infty}\frac{h\rho\, d\rho\, d\phi}{\left[\rho^2 + h^2\right]^{3/2}}\hat{a}_z$$

$$= \frac{\rho_s h}{4\pi\varepsilon_0}2\pi\int_0^\infty\left[\rho^2 + h^2\right]^{-3/2}\frac{1}{2}d\left(\rho^2\right)\hat{a}_z$$

$$= \frac{\rho_s h}{2\varepsilon_0}\left\{-\left[\rho^2 + h^2\right]^{-1/2}\right\}_0^\infty\hat{a}_z$$

$$= \frac{\rho_s}{2\varepsilon_0}\hat{a}_z$$



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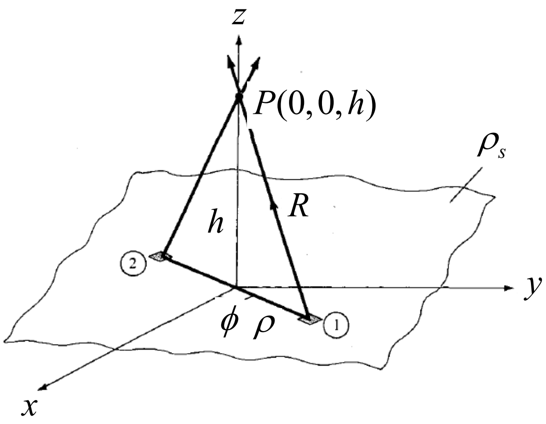
Surface Charge

In general:

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_n$$

Electric field is independent of distance from the sheet of charge.



Parallel Plate Capacitor

$$\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{a}_n + \frac{-\rho_s}{2\varepsilon_0}(-\hat{a}_n)$$

$$= \frac{\rho_s}{\varepsilon_0}\hat{a}_n$$