ELECTROSTATICS

1

1

Electric Charge

- Two types of electric charge: positive and negative.
- Unit of charge is the coulomb (C).
- Charge of electron (negative) or proton (positive) is
  $\pm e$, $e = 1.602 \times 10^{-19}$ C.
- Charge is quantized
  $Q = \pm Ne$.

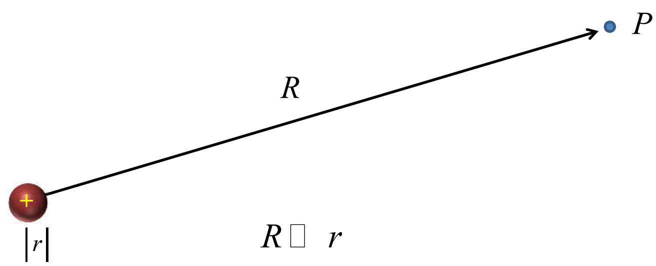
2

2

Point Charge

Charge that is located in a body whose dimensions are much smaller than other relevant dimensions.

Example: Collection of electric charges on a pinhead.



$R \gg r$

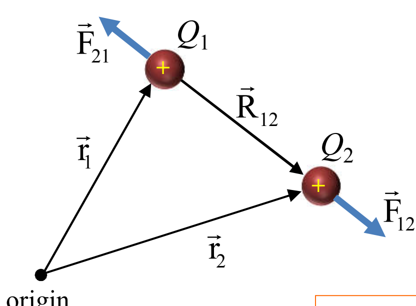
3

3

Coulomb's Law

The force between two point charges Q_1 and Q_2 is

1. Along the line joining them
2. Directly proportional to product Q_1Q_2
3. Inversely proportional to the square of the distance between them.



$$F = \frac{k\,Q_1Q_2}{R^2}$$

In SI units Q_1, Q_2: coulombs
R: meters
F: newtons

$$k = \frac{1}{4\pi\varepsilon_0}, \quad \varepsilon_0 = 8.854\times10^{-12}\ \text{F/m}$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}$$

4

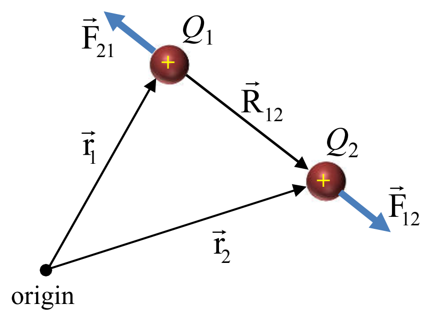
4

Electric Force

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^3}\vec{R}_{12}$$

$$\vec{F}_{12} = \frac{Q_1 Q_2 \left(\vec{r}_2 - \vec{r}_1\right)}{4\pi\varepsilon_0 \left|\vec{r}_2 - \vec{r}_1\right|^3}$$

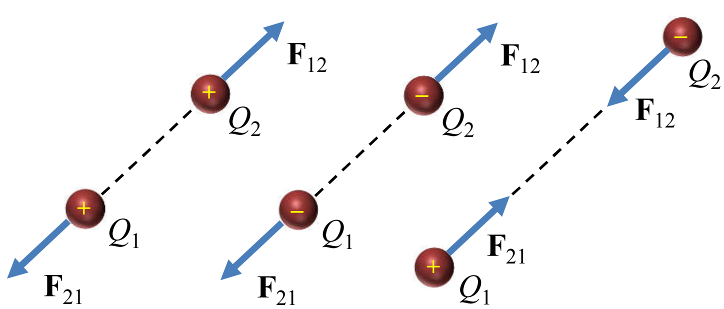


Q_1 and Q_2 MUST be point charges!

5

5

Electric Force

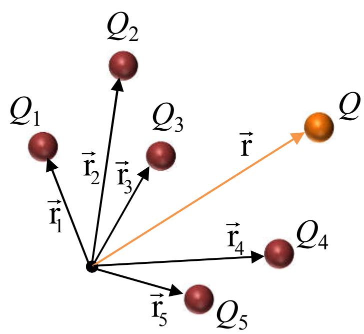


Like charges repel and opposite charges attract!!

6

6

Principle of Superposition



$$\vec{F} = \frac{QQ_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_1|^3} + \frac{QQ_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_2|^3} + \cdots + \frac{QQ_5(\vec{r}-\vec{r}_5)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_5|^3}$$

7

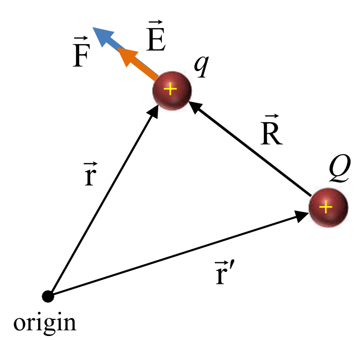
7

Electric Field

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$= \frac{Q(\vec{r}-\vec{r}')}{4\pi\varepsilon_0 |\vec{r}-\vec{r}'|^3}$$

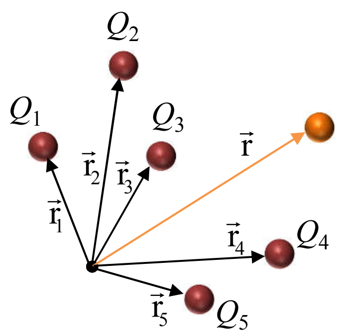


Unit: Newton/Coulomb, same as Volt/meter

8

8

Principle of Superposition



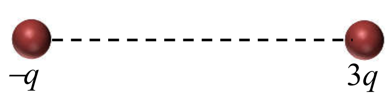
$$\vec{E} = \frac{Q_1(\vec{r}-\vec{r}_1)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_1|^3} + \frac{Q_2(\vec{r}-\vec{r}_2)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_2|^3} + \cdots + \frac{Q_5(\vec{r}-\vec{r}_5)}{4\pi\varepsilon_0 |\vec{r}-\vec{r}_5|^3}$$

9

9

Concept Question: Electric Field

Two objects with charges $-q$ and $+3q$ are placed on a line as shown.



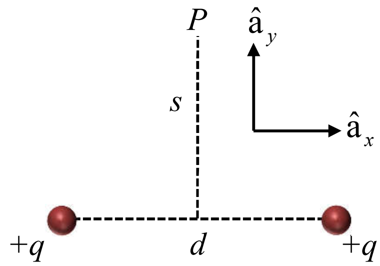
Beside in infinity, where else can electric field be zero?

1. Between the two charges
2. To the right of the charge on the right
3. To the left of the charge on the left
4. The electric field is only zero at infinity away from the charges.

10

10

Concept Question: 2 Equal Charges



E field at *P* is:

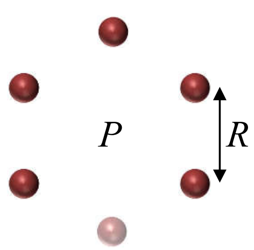
1. $\vec{E} = \dfrac{2kqs}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\hat{a}_y$

2. $\vec{E} = \dfrac{2kqd}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\hat{a}_y$

3. $\vec{E} = -\dfrac{2kqd}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\hat{a}_x$

4. $\vec{E} = -\dfrac{2kqs}{\left[s^2 + \dfrac{d^2}{4}\right]^{3/2}}\hat{a}_x$

Concept Question: 6 Equal Charges



Six equal positive charges *q* sit at the vertices of a regular hexagon with sides of length *R*. We remove the bottom charge. The electric field at the center of the hexagon (at point *P*) is:

1. $\vec{E} = \dfrac{2kq}{R^2}\hat{a}_y$

2. $\vec{E} = -\dfrac{2kq}{R^2}\hat{a}_y$

3. $\vec{E} = \dfrac{kq}{R^2}\hat{a}_y$

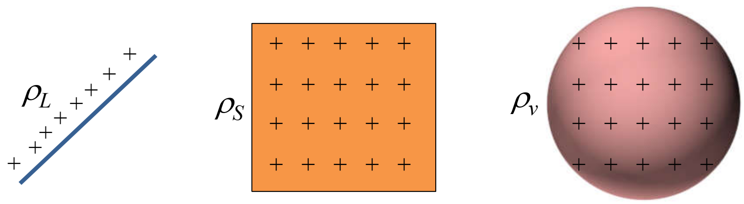
4. $\vec{E} = -\dfrac{kq}{R^2}\hat{a}_y$

5. $\vec{E} = 0$

Continuous Charge Distribution

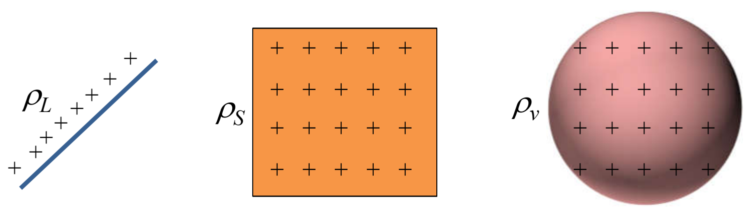
So far we have considered forces and electric fields due to point charges only. Now we consider continuous charge distribution along a line, on a surface, or in a volume.

Point charge



Line charge   Surface charge   Volume charge

Continuous Charge Distribution



Line charge   Surface charge   Volume charge

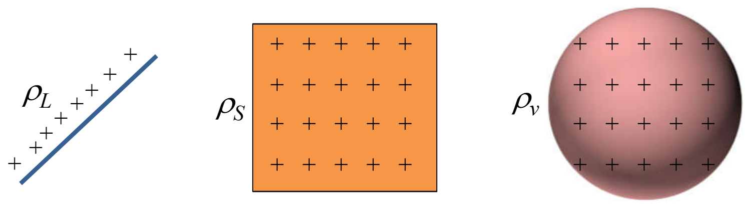
Total Charge *Q*:

$$dQ = \rho_L \, dl \rightarrow Q = \int_L \rho_L \, dl \qquad \text{(Line charge)}$$

$$dQ = \rho_S \, dS \rightarrow Q = \int_S \rho_S \, dS \qquad \text{(Surface charge)}$$

$$dQ = \rho_v \, dv \rightarrow Q = \int_v \rho_v \, dv \qquad \text{(Volume charge)}$$

Continuous Charge Distribution



Line charge Surface charge Volume charge

Electric Field:

$$\vec{E} = \int \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2} \hat{a}_R \quad \text{(Line charge)}$$

$$\vec{E} = \int \frac{\rho_S \, dS}{4\pi\varepsilon_0 R^2} \hat{a}_R \quad \text{(Surface charge)}$$

$$\vec{E} = \int \frac{\rho_v \, dv}{4\pi\varepsilon_0 R^2} \hat{a}_R \quad \text{(Volume charge)}$$

15

15

Line Charge

Total charge:

$$dQ = \rho_L \, dl = \rho_L \, dz$$

$$Q = \int_{z_A}^{z_B} \rho_L \, dz$$

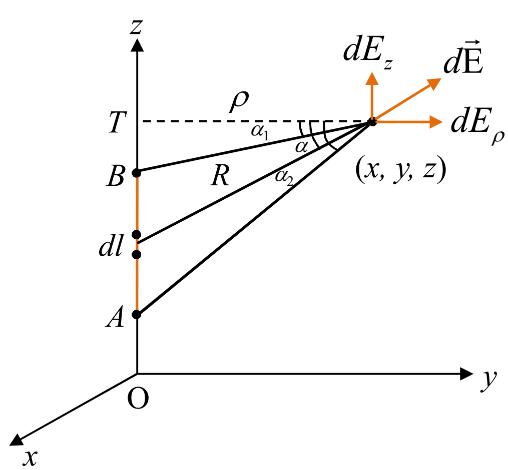
Distance and Direction:

$$\vec{R} = (x, y, z) - (0, 0, z')$$

$$= x\,\hat{a}_x + y\,\hat{a}_y + (z - z')\,\hat{a}_z$$

$$\vec{R} = \rho\,\hat{a}_\rho + (z - z')\,\hat{a}_z$$

$$\frac{\hat{a}_R}{R^2} = \frac{\vec{R}}{|\vec{R}|^3} = \frac{\rho\,\hat{a}_\rho + (z - z')\,\hat{a}_z}{\left[\rho^2 + (z - z')^2\right]^{3/2}}$$



16

16

Line Charge

From Coulomb's law:

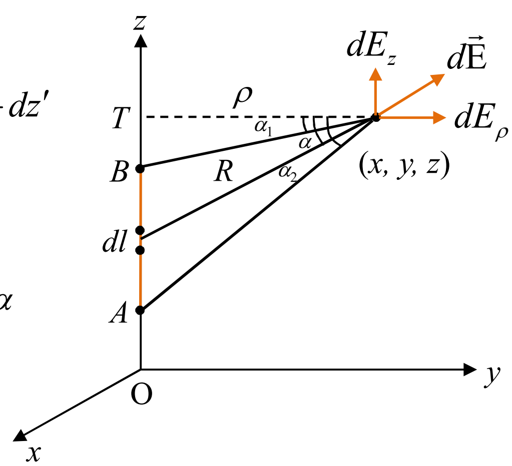
$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0}\int \frac{\rho\,\hat{a}_\rho + (z-z')\,\hat{a}_z}{\left[\rho^2 + (z-z')^2\right]^{3/2}}\,dz'$$

Integrate:

$$R = \left[\rho^2 + (z-z')^2\right]^{1/2} = \rho\sec\alpha$$

$$z' = OT - \rho\tan\alpha$$

$$dz' = -\rho\sec^2\alpha\,d\alpha$$



17

17

Line Charge

Integrate:

$$\vec{E} = \frac{-\rho_L}{4\pi\varepsilon_0}\int_{\alpha_1}^{\alpha_2} \frac{\rho\sec^2\alpha\left[\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z\right]d\alpha}{\rho^2\sec^2\alpha}$$

$$= -\frac{\rho_L}{4\pi\varepsilon_0\rho}\int_{\alpha_1}^{\alpha_2}\left[\cos\alpha\,\hat{a}_\rho + \sin\alpha\,\hat{a}_z\right]d\alpha$$

For a finite line charge,

$$\vec{E} = \frac{\rho_L}{4\pi\varepsilon_0\rho}\left[-(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\,\hat{a}_z\right]$$

Special case:

For an infinite line charge, $\alpha_1 = \pi/2,\ \alpha_2 = -\pi/2$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\,\hat{a}_\rho}$$

18

18