WAVE POLARIZATION

1

Wave Polarization

The polarization describes the shape and locus of the tip of the *E* field at a given point in space as a function of time.

$$\vec{E}_s(z) = \hat{a}_x E_{sx}(z) + \hat{a}_y E_{sy}(z)$$

$$E_{sx}(z) = E_{x0} e^{-\gamma z} = x e^{-j\beta z}$$

$$E_{sy}(z) = E_{y0} e^{-\gamma z} = y e^{j\phi} e^{-j\beta z}$$

$$\vec{E}_s(z) = \hat{a}_x x e^{-j\beta z} + \hat{a}_y y e^{j\phi} e^{-j\beta z} = \left(\hat{a}_x x + \hat{a}_y y e^{j\phi}\right) e^{-j\beta z}$$

2

Instantaneous Field

$$\vec{E}(z,t) = \text{Re}\left(\vec{E}_s(z)e^{j\omega t}\right)$$
$$= \hat{a}_x x\cos(\omega t - \beta z) + \hat{a}_y y\cos(\omega t - \beta z + \phi)$$

The magnitude:

$$|\vec{E}(z,t)| = \left[E_x^2(z,t) + E_y^2(z,t)\right]^{1/2}$$
$$= \left[x^2\cos^2(\omega t - \beta z) + y^2\cos^2(\omega t - \beta z + \phi)\right]^{1/2}$$

The direction:

$$\psi(z,t) = \tan^{-1}\left(\frac{E_y(z,t)}{E_x(z,t)}\right)$$

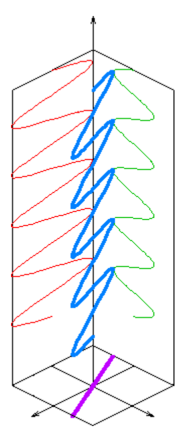
In general, $|E(z,t)|$ and $\psi(z,t)$ are functions of z and t.

3

Linear Polarization

A wave is said to be linearly polarized if $E_x(z,t)$ and $E_y(z,t)$ are in phase (i.e., $\phi = 0$) or out of phase ($\phi = \pi$).

$$\vec{E}(0,t) = \left(\hat{a}_x x + \hat{a}_y y\right)\cos\omega t \quad \text{(in phase)}$$
$$\vec{E}(0,t) = \left(\hat{a}_x x - \hat{a}_y y\right)\cos\omega t \quad \text{(out-of-phase)}$$



Consider the out-of-phase case.

$$|\vec{E}(0,t)| = \left[x^2 + y^2\right]^{1/2}\cos\omega t$$

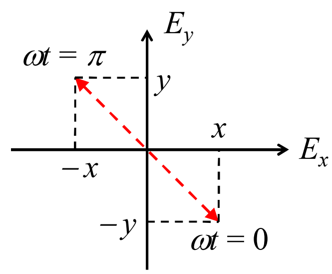
$$\psi(z,t) = \tan^{-1}\left(\frac{-y}{x}\right)$$

4

Linear Polarization

$$|\vec{E}(0,t)| = \left[x^2 + y^2\right]^{1/2} \cos \omega t$$

$$\psi = \tan^{-1}\left(\frac{-y}{x}\right)$$



ψ is independent of both z and t.

5

Circular Polarization

Let magnitudes of $E_x(z,t)$ and $E_y(z,t)$ are equal and the phase difference $\phi = \pm\, \pi/2$.

If $x = y = a$ and $\phi = \pi/2$

$$\vec{E}_s(z) = \left(\hat{a}_x a + \hat{a}_y a e^{j\pi/2}\right) e^{-j\beta z}$$
$$= a\left(\hat{a}_x + j\,\hat{a}_y\right) e^{-j\beta z}$$

6

Instantaneous Field

$$\vec{E}(z,t) = \mathrm{Re}\left(\vec{E}_s(z)e^{j\omega t}\right) = \hat{a}_x a\cos(\omega t - \beta z) + \hat{a}_y a\cos(\omega t - \beta z + \pi/2)$$
$$= \hat{a}_x a\cos(\omega t - \beta z) - \hat{a}_y a\sin(\omega t - \beta z)$$

The magnitude:

$$|\vec{E}(z,t)| = \left[E_x^2(z,t) + E_y^2(z,t)\right]^{1/2}$$
$$= \left[a^2\cos^2(\omega t - \beta z) + a^2\sin^2(\omega t - \beta z)\right]^{1/2} = a$$

The direction:

$$\psi(z,t) = \tan^{-1}\left(\frac{E_y(z,t)}{E_x(z,t)}\right) = \tan^{-1}\left(\frac{-a\sin(\omega t - \beta z)}{a\cos(\omega t - \beta z)}\right) = -(\omega t - \beta z)$$

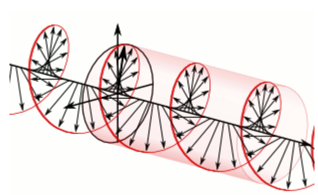
$|E|$ is independent of z and t, whereas ψ depends on both z and t.

Left-Hand Circular Polarization

If $x = y = a$ and $\phi = \pi/2$

$$|\vec{E}(z,t)| = a$$

$$\psi(z,t) = -(\omega t - \beta z)$$



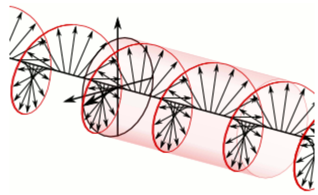
E field rotates clockwise.

Right-Hand Circular Polarization

If $x = y = a$ and $\phi = -\pi/2$

$$|\vec{\mathrm{E}}(z,t)| = a$$

$$\psi(z,t) = (\omega t - \beta z)$$



E field rotates counter-clockwise.

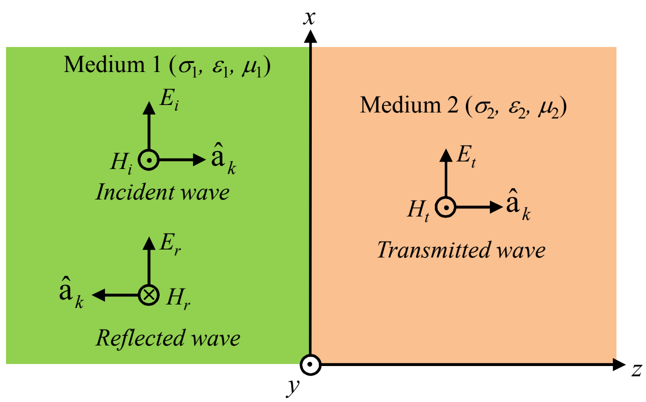
9

REFLECTION AND TRANSMISSION OF A PLANE WAVE

10

Normal Incidence

- EM wave incident on an interface between two media may be reflected or transmitted depending on the constitutive parameters.
- We assume that the incident EM wave is normal to the boundary.



11

Normal Incidence

Incident Wave:

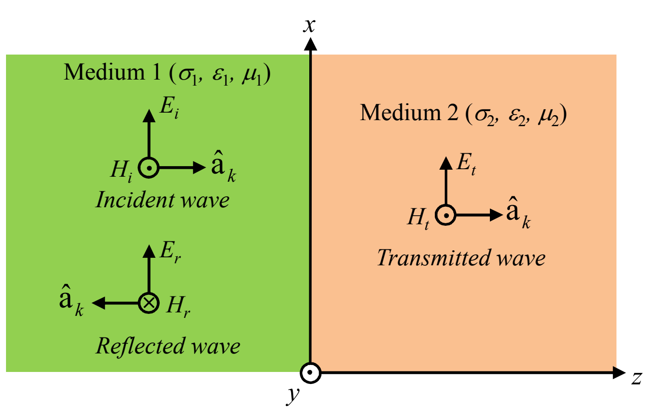
$$\vec{E}_{is}(z) = E_{i0} e^{-\gamma_1 z}\,\hat{a}_x$$

$$\vec{H}_{is}(z) = H_{i0} e^{-\gamma_1 z}\,\hat{a}_y = \frac{E_{i0}}{\eta_1} e^{-\gamma_1 z}\,\hat{a}_y$$

Reflected Wave:

$$\vec{E}_{rs}(z) = E_{r0} e^{\gamma_1 z}\,\hat{a}_x$$

$$\vec{H}_{rs}(z) = -H_{r0} e^{\gamma_1 z}\,\hat{a}_y = -\frac{E_{r0}}{\eta_1} e^{\gamma_1 z}\,\hat{a}_y$$



12

Normal Incidence

Transmitted Wave:

$$\vec{E}_{ts}(z) = E_{t0} e^{-\gamma_2 z}\, \hat{a}_x$$

$$\vec{H}_{ts}(z) = H_{t0} e^{-\gamma_2 z}\, \hat{a}_y = \frac{E_{t0}}{\eta_2} e^{-\gamma_2 z}\, \hat{a}_y$$

Total Fields:

Medium 1:

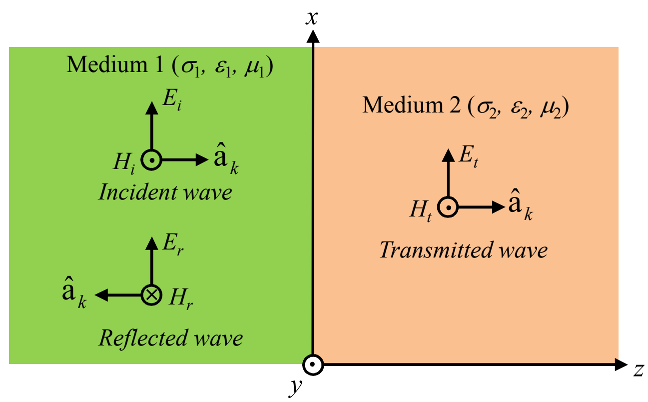
$$\vec{E}_1 = \vec{E}_i + \vec{E}_r$$

$$\vec{H}_1 = \vec{H}_i + \vec{H}_r$$

Medium 2:

$$\vec{E}_2 = \vec{E}_t$$

$$\vec{H}_2 = \vec{H}_t$$



Reflection and Transmission

Transverse waves:

At interface $\rightarrow$ $\vec{E}_{1\tan} = \vec{E}_{2\tan}$ and $\vec{H}_{1\tan} = \vec{H}_{2\tan}$

$$\vec{E}_i(0) + \vec{E}_r(0) = \vec{E}_t(0) \rightarrow E_{i0} + E_{r0} = E_{t0}$$

$$\vec{H}_i(0) + \vec{H}_r(0) = \vec{H}_t(0) \rightarrow \frac{1}{\eta_1}\left(E_{i0} - E_{r0}\right) = \frac{E_{t0}}{\eta_2}$$

$$E_{r0} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_{i0} = \Gamma E_{i0} \qquad \Gamma = \frac{E_{r0}}{E_{i0}}$$ $\rightarrow$ Reflection coefficient

$$E_{t0} = \frac{2\eta_2}{\eta_2 + \eta_1} E_{i0} = \tau E_{i0} \qquad \tau = \frac{E_{t0}}{E_{i0}}$$ $\rightarrow$ Transmission coefficient

Perfect Dielectric-Perfect Conductor

$$\sigma_1 = 0$$
$$\sigma_2 \ \square \ \infty$$

$$|\eta_2| = \frac{\sqrt{\mu_2 / \varepsilon_2}}{\sqrt{1+\left(\dfrac{\sigma_2}{\omega\varepsilon_2}\right)^2}} \approx \frac{\omega}{v_p \sigma_2} \approx 0$$

$$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1 \qquad \tau = 0$$

Wave is totally reflected from the interface.

Standing Waves

Medium 1: $\sigma_1 = 0 \Rightarrow \alpha_1 = 0, \gamma_1 = j\beta_1$

$$\vec{E}_{1s} = \vec{E}_{is} + \vec{E}_{rs} = \left(E_{i0}e^{-\gamma_1 z} + E_{r0}e^{\gamma_1 z}\right)\hat{a}_x$$

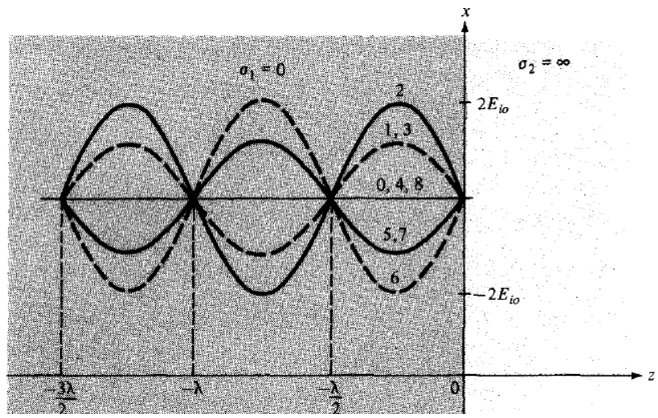
$$E_{1s} = -E_{i0}\left(e^{j\beta_1 z} - e^{-j\beta_1 z}\right)\hat{a}_x = -2jE_{i0}\sin\beta_1 z\,\hat{a}_x$$

$$\vec{E}_1 = \mathrm{Re}\left(\vec{E}_{1s}e^{j\omega t}\right) = 2E_{i0}\sin\beta_1 z\sin\omega t\,\hat{a}_x$$

$$\vec{H}_1 = \frac{2E_{i0}}{\eta_1}\cos\beta_1 z\cos\omega t\,\hat{a}_y$$

Standing Wave

$$\vec{E}_1 = 2E_{i0}\sin\beta_1 z\sin\omega t\,\hat{a}_x$$



17