ELECTROMAGNETIC WAVE PROPAGATION

1

Lossless Dielectric

$$\sigma \ll \omega\varepsilon$$

$$\sigma \approx 0, \varepsilon = \varepsilon_0\varepsilon_r, \mu = \mu_0\mu_r$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]} = 0 \qquad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]} = \omega\sqrt{\mu\varepsilon}$$

$$\eta = j\frac{\omega\mu}{\gamma} = \sqrt{\frac{\mu}{\varepsilon}}\angle 0^\circ$$

***E* and *H* are in time phase with each other.**

$$\vec{\mathrm{E}}(z,t) = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{\mathrm{a}}_x = E_0\cos(\omega t - \beta z)\,\hat{\mathrm{a}}_x$$

$$\vec{\mathrm{H}}(z,t) = (E_0/\eta)e^{-\alpha z}\cos(\omega t - \beta z)\,\hat{\mathrm{a}}_x = \sqrt{(\varepsilon/\mu)}E_0\cos(\omega t - \beta z)\,\hat{\mathrm{a}}_x$$

2

Free Space: Intrinsic Impedance

$$\sigma \approx 0, \varepsilon = \varepsilon_0, \mu = \mu_0$$

$$\alpha = 0, \beta = \omega\sqrt{\mu_0 \varepsilon_0} = \frac{\omega}{c}$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.854 \times 10^{-12}}} = 120\pi \approx 377\,\Omega$$

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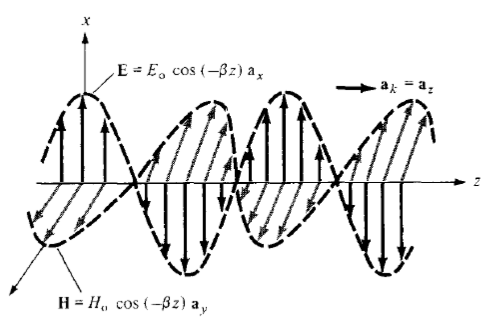
EM Waves in Free Space

$$\vec{E} = E_0 \cos(\omega t - \beta z)\,\hat{a}_x$$

$$\vec{H} = H_0 \cos(\omega t - \beta z)\,\hat{a}_y = \frac{E_0}{\eta_0}\cos(\omega t - \beta z)\,\hat{a}_y$$

$$\hat{a}_E \times \hat{a}_H = \hat{a}_k \rightarrow \text{TEM waves}$$



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EM Waves in Good Conductors

$$\sigma \approx \infty, \varepsilon = \varepsilon_0 \varepsilon_r, \mu = \mu_0 \mu_r$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]} \qquad \beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

***E* leads *H* by 45°.**

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EM Waves in Good Conductors

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{a}_x$$

$$\vec{H} = \frac{E_0}{\sqrt{\omega\mu / \sigma}} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\,\hat{a}_y$$

Amplitude is attenuated by the factor $e^{-\alpha z}$.

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ELECTROMAGNETIC WAVES AND CURRENT IN CONDUCTORS

7

EM Waves in Good Conductors

$$\sigma \,\square\, \infty$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]} \qquad \beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^{\circ}$$

Electric field leads magnetic field by 45^{o}.

8

EM Waves in Good Conductors

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{a}_x$$

$$\vec{H} = \frac{E_0}{\sqrt{\omega\mu/\sigma}} e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\,\hat{a}_y$$

Amplitude is attenuated by the factor $e^{-\alpha z}$.

9

Skin Depth

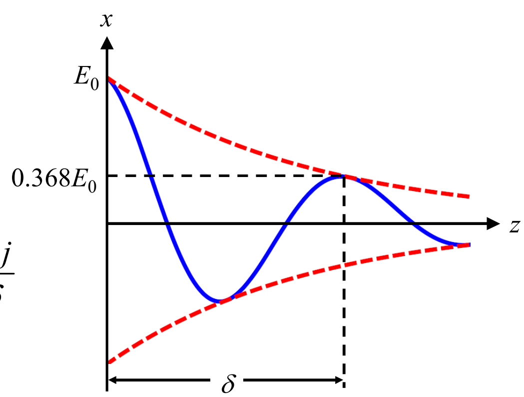
Skin depth, δ: Wave amplitude decreases by a factor e^{-1} (~ 37%).

$$E_0 e^{-\alpha\delta} = E_0 e^{-1}$$

$$\delta = \frac{1}{\alpha} \Rightarrow \delta = \sqrt{\frac{2}{\omega\mu\sigma}}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} e^{j\pi/4} = \frac{\sqrt{2}}{\sigma\delta} e^{j\pi/4} = \frac{1+j}{\sigma\delta}$$

$$\vec{E} = E_0 e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right)\hat{a}_x$$



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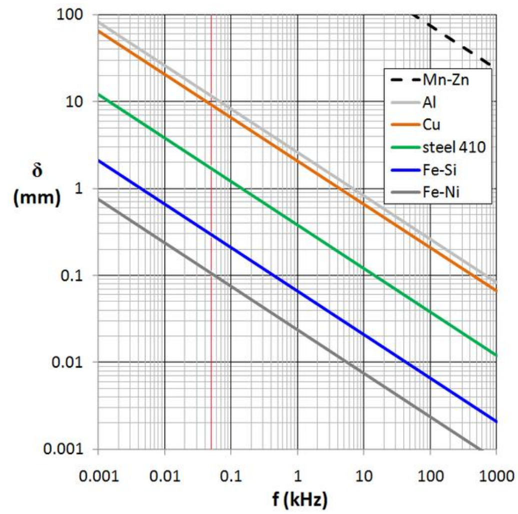
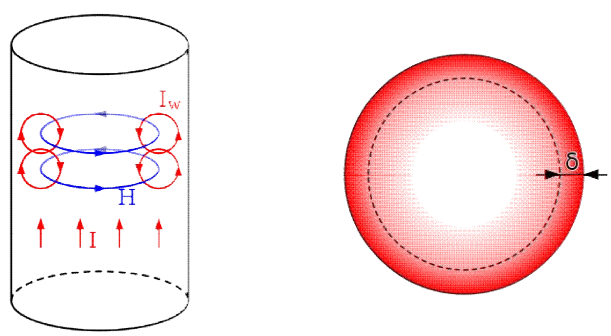
Skin Depth vs. Frequency



Skin effect is used as electromagnetic shielding!

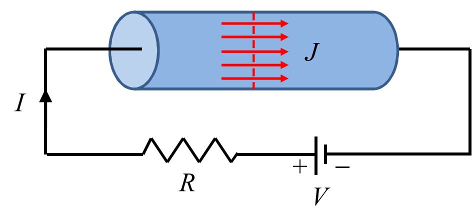
Skin Effect



- Skin depth is due to the circulating eddy currents (arising from a changing H field) cancelling the current flow in the center of a conductor and reinforcing it in the skin.
- Distribution of current flow in a cylindrical conductor, shown in cross section. For alternating current, most (63%) of the electrical current flows between the surface and the skin depth, *δ*.

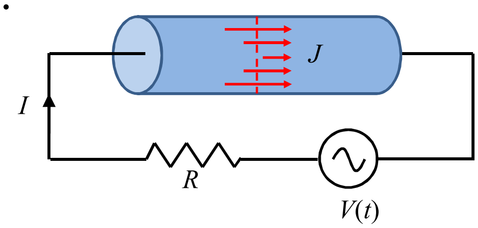
Current Flow in a Good Conductor

DC:



Uniform current density

AC:



Non-uniform current density

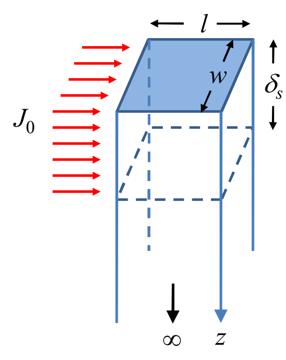
AC Resistance

$$R_{\text{dc}} = \frac{l}{\sigma S}$$

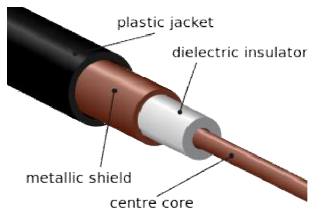
$$\eta = \frac{1}{\sigma\delta} + \frac{j}{\sigma\delta} = R_s + jX_s$$

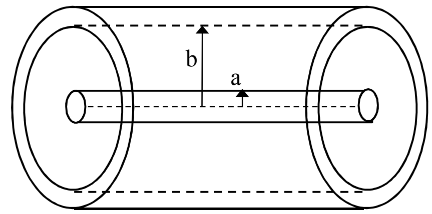
$$R_{\text{ac}} = \frac{l}{\sigma\delta w} = R_s \frac{l}{w}$$

R_{ac} is equivalent to R_{dc} of a plane conductor of length l and cross section $A = \delta w$.



Co-Axial Cable





The current flowing through the co-axial cable is concentrated within a thin layer on the outer surface of the inner conductor and on the inner surface of the outer conductor.

Equivalent inner conductor

Equivalent outer conductor