ELECTROMAGNETIC WAVES

Uniform Plane Wave Solutions

The 1-D wave equation

$$\frac{\partial^2 E_y}{\partial z^2} - \epsilon_o \mu_o \frac{\partial^2 E_y}{\partial t^2} = 0$$

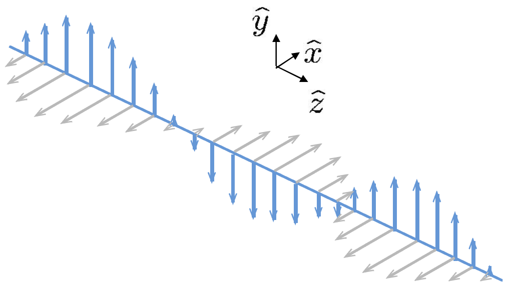
- $E_y(z,t)$ is any function for which the second derivative in space equals its second derivative in time, times a constant. The solution is therefore any function with the same dependence on time as on space, e.g.

$$E_y = f_+(t - z/c) + f_-(t + z/c)$$

- The functions $f_+(z\text{-}ct)$ and $f_-(z+ct)$ represent uniform waves propagating in the $+z$ and $-z$ directions respectively.

Magnetic Field of a Uniform Plane Wave

$$\frac{\partial B_x(z_o)}{\partial z} = \epsilon_o \mu_o \frac{\partial E_y}{\partial t}$$



In vacuum…

$$E_y = f_+(t - z/c) + f_-(t + z/c)$$

$$H_x = -\sqrt{\frac{\epsilon_o}{\mu_o}}\Big(f_+(t - z/c) - f_-(t + z/c)\Big)$$

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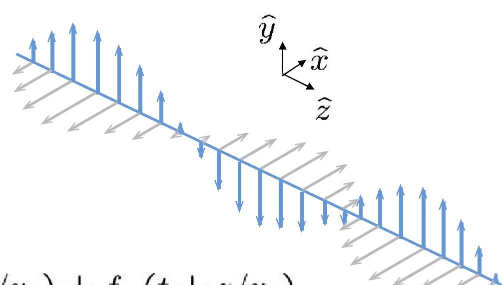
A Uniform Plane Wave

$$\frac{\partial B_x(z_o)}{\partial z} = \epsilon \mu \frac{\partial E_y}{\partial t}$$



Inside a material…

$$E_y = f_+(t - z/v_p) + f_-(t + z/v_p)$$

$$H_x = -\sqrt{\frac{\epsilon}{\mu}}\Big(f_+(t - z/v_p) - f_-(t + z/v_p)\Big)$$

… where $\boxed{v_p = \frac{1}{\sqrt{\mu\epsilon}}}$ is known as the phase velocity of the wave

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The Characteristic Impedance

$$H_x = -\sqrt{\frac{\epsilon}{\mu}}\left(f_+(t - z/v_p) - f_-(t + z/v_p)\right)$$

- *η* is the ***intrinsic impedance*** of the medium given by

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

- Like the velocity of propagation, the intrinsic impedance is independent of the source and is determined only by the properties of the medium.

$$\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} \approx 377 \text{ Ohms}$$

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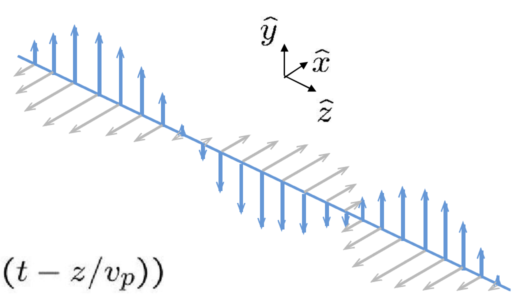
Sinusoidal Uniform Plane Waves



$$f_+(t - z/v_p) = A\cos\left(\omega\left(t - z/v_p\right)\right)$$

$$= A\cos\left(\omega t - kz\right)$$ … where $k = \frac{\omega}{v_p}$ … is known as the wave-number

$$f_-(t + z/v_p) = A\cos\left(\omega\left(t + z/v_p\right)\right)$$

$$= A\cos\left(\omega t + kz\right)$$

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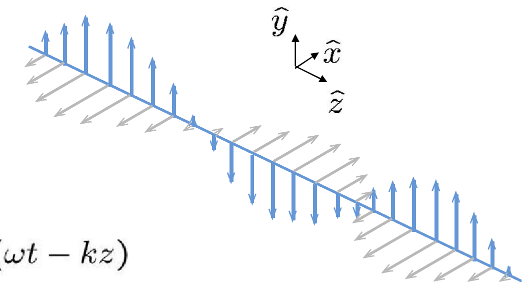
Sinusoidal Uniform Plane Waves



$$f_+(t - z/c) = A\cos(\omega t - kz)$$

$$f_-(t + z/c) = A\cos(\omega t + kz) \qquad k = \frac{\omega}{c}$$

$$E_y = A_1 cos(\omega t - kz) + A_2 cos(\omega t + kz)$$

$$H_x = -\frac{A_1}{\eta} cos(\omega t - kz) + \frac{A_2}{\eta} cos(\omega t + kz)$$

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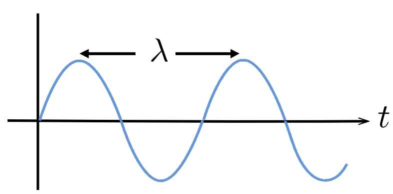
Sinusoidal Uniform Plane Waves

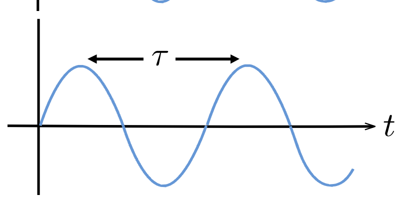
$$k = \frac{\omega}{v_p}$$

Spatial quantities:



$$k = \frac{2\pi}{\lambda}$$

Temporal quantities:



$$\omega = \frac{2\pi}{\tau}$$

$$E_y = A_1 \, cos(\omega t - kz) + A_2 \, cos(\omega t + kz)$$

$$H_x = -\frac{A_1}{\eta} \, cos(\omega t - kz) + \frac{A_2}{\eta} \, cos(\omega t + kz)$$

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