TIME-VARYING FIELDS

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EMF

$$\psi = \int_S \vec{B} \cdot d\vec{S}$$

Variation of flux can be caused in three ways:

1. By having a stationary loop in a time-varying B field
2. By having a time-varying loop area in a static B field
3. By having a time-varying loop area in a time-varying B field.

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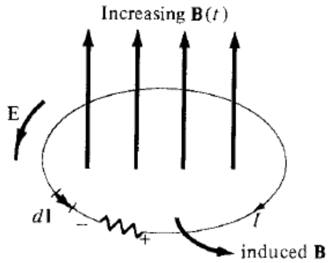
Transformer EMF

Stationary loop in varying B field:

$$V_{\text{emf}} = \oint_L \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$



Using Stoke's theorem

$$\int_S (\nabla \times \vec{E}) \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ → Maxwell's equation for time-varying field

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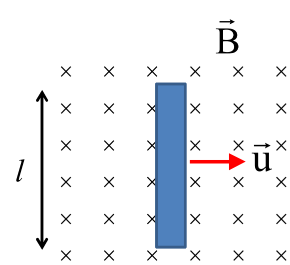
Motional EMF

- Consider a conducting bar of length l moving through a uniform magnetic field which points into the page.
- Charges inside the bar experience a magnetic force F_m.

$$\vec{F}_m = Q\vec{u} \times \vec{B}$$

- Positive and negative charges accumulate at the opposite ends.



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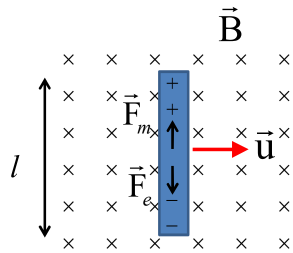
Motional EMF

→ Separation of charge gives rise to electric field and electric force.



$\vec{F}_e = Q\vec{E}_m$ $\quad E_m$: motional electric field

$$\vec{F}_e = \vec{F}_m \Rightarrow \vec{E}_m = \frac{\vec{F}_m}{Q} = \vec{u} \times \vec{B}$$

$$V_{\text{emf}} = \int_L \vec{E}_m \cdot d\vec{l} = \int_L (\vec{u} \times \vec{B}) \cdot d\vec{l} = uBl$$

Motional emf is found in motors and generators.

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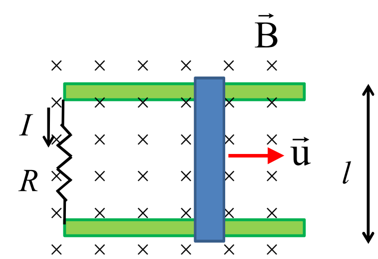
Motional EMF

Now consider that the conducting bar slides along two frictionless conducting rails that are at a distance l apart and connected together by a resistor with resistance R.

$$\psi = BS = Blx$$

$$V_{\text{emf}} = -\frac{d\psi}{dt} = -Bl\frac{dx}{dt} = -Blu$$



$$I = \frac{|V_{\text{emf}}|}{R} = \frac{uBl}{R}$$

$$\vec{F}_m = Q\vec{u} \times \vec{B} = I\vec{l} \times \vec{B} \Rightarrow F_m = IlB$$

F_m is in the opposite direction of u.

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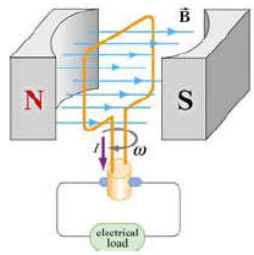
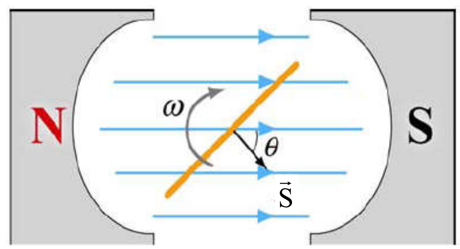
Generator



$$\psi = BS\cos\theta = BS\cos\omega t$$

$$\frac{d\psi}{dt} = -BS\omega\sin\omega t$$

$$V_{\text{emf}} = -N\frac{d\psi}{dt} = NBS\omega\sin\omega t$$



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Varying Both B Field and Area S

Both transformer emf and motional emf are present.

$$V_{\text{emf}} = \int_L \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial B}{\partial t}\cdot dS + \int_L (\vec{u}\times\vec{B})\cdot d\vec{l}$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} + \nabla\times(\vec{u}\times\vec{B})$$

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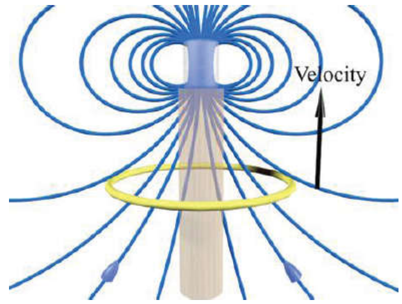
Concept Question: Faraday's Law

A coil moves up from underneath a magnet with its north pole pointing upward. The current in the coil and the force on the coil:

1. Current clockwise; force up
2. Current counterclockwise; force up
3. Current clockwise; force down
4. Current counterclockwise; force down.



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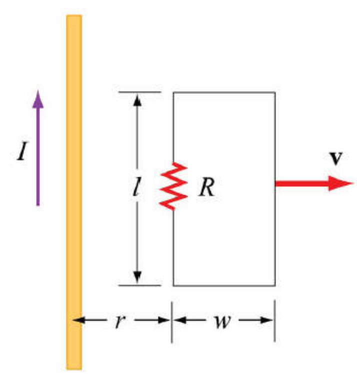
Concept Question: Rectangular Loop

A circuit in the form of a rectangular piece of wire is pulled away from a long wire carrying current I in the direction shown. The induced current is

1. Clockwise
2. Counterclockwise
3. Neither, current is zero
4. Can't be determined



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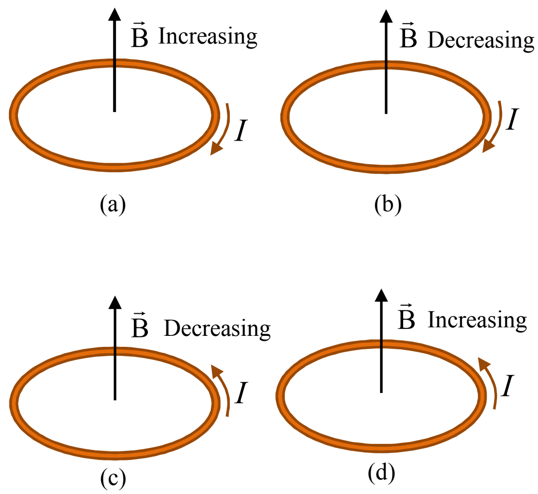
Concept Question: Rectangular Loop

Assuming that each loop is stationary and the time-varying magnetic field B induces current *I*, which of the configurations in Figure are incorrect?



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