MAGNETIC POTENTIAL

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Magnetic Potential

Electrostatic $\rightarrow$ potential

$$\vec{E} = -\nabla V$$

- Can we define magnetic potential?
- Magnetic field is different from electric field
  - Force on a charge in an electric field depends on only the position of the charge.
  - Force on a charge in a magnetic field depends on both the position and direction of motion of the charge.

Magnetic potential can be both scalar and vector.

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Scalar Magnetic Potentials

Vector identity:

$$\nabla \times (\nabla V) = 0$$

$$\vec{H} = -\nabla V_m$$

$$\nabla \times \vec{H} = \vec{J}$$

$$\Downarrow$$

$$\vec{J} = 0.$$

Permanent magnet.

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Vector Magnetic Potentials

Vector identity:

$$\nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\Downarrow$$

$$\vec{B} = \nabla \times \vec{A}$$

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Vector Magnetic Potentials

Electrostatic: $V = \int \frac{dQ}{4\pi\varepsilon_0 r}$

Magnetostatic:

$$\vec{A} = \int_L \frac{\mu_0 I \, d\vec{l}}{4\pi R} \quad \text{for line current}$$

$$\vec{A} = \int_L \frac{\mu_0 \vec{K} \, dS}{4\pi R} \quad \text{for surface current}$$

$$\vec{A} = \int_L \frac{\mu_0 \vec{J} \, d\vec{l}}{4\pi R} \quad \text{for volume current}$$

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Magnetic Flux

$$\psi = \int_S \vec{B} \cdot d\vec{S}$$

$$= \int_S (\nabla \times \vec{A}) \cdot d\vec{S} = \oint_L \vec{A} \cdot d\vec{l}$$

Vector potential provides an approach to solving EM problems!

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MAGNETIC FORCES

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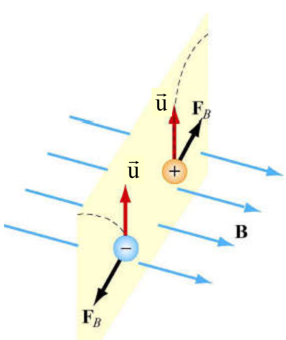
Moving Charge

Electrostatic $\rightarrow$ $\vec{F}_e = Q\vec{E}$

Magnetostatic $\rightarrow$ $\vec{F}_m = Q\vec{u} \times \vec{B}$



- Magnetic force is perpendicular to both the velocity of the charge and the magnetic field.
- The magnetic force acts on the charge only when it is in motion.
- Work done:

$$dW = \vec{F}_m \cdot d\vec{l} = (\vec{F}_m \cdot \vec{u})dt = 0$$

- F_m changes the direction of motion but not the speed.

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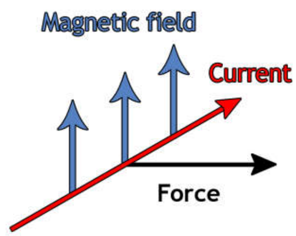
Current Element

$$I d\vec{l} = \frac{dQ}{dt} d\vec{l} = dQ \frac{d\vec{l}}{dt} = dQ\,\vec{u}$$

$$I d\vec{l} \equiv \vec{K} dS \equiv \vec{J} dv$$



Force:

$$d\vec{F} = I d\vec{l} \times \vec{B}, \quad \vec{F} = \oint_L I d\vec{l} \times \vec{B} \quad \rightarrow \text{Line current element}$$

$$d\vec{F} = \vec{K} dS \times \vec{B}, \quad \vec{F} = \int_S \vec{K} dS \times \vec{B} \quad \rightarrow \text{Surface current element}$$

$$d\vec{F} = \vec{J} dv \times \vec{B}, \quad \vec{F} = \int_v \vec{J} dv \times \vec{B} \quad \rightarrow \text{Volume current element}$$

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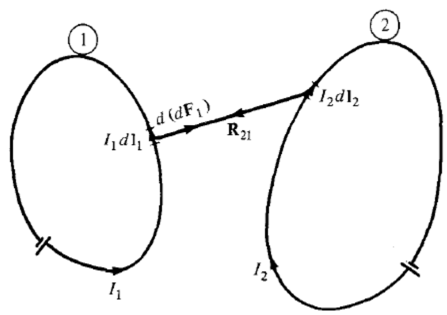
Two Current Elements

$$d(d\vec{F}_1) = I_1 d\vec{l}_1 \times d\vec{B}_2$$

$$d\vec{B}_2 = \frac{\mu_0 I_2 d\vec{l}_2 \times \hat{a}_{R_{21}}}{4\pi R_{21}^2}$$

$$d(d\vec{F}_1) = \frac{\mu_0 I_1 d\vec{l}_1 \times (I_2 d\vec{l}_2 \times \hat{a}_{R_{21}})}{4\pi R_{21}^2}$$



Total force on loop 1 due to loop 2

$$\vec{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\vec{l}_1 \times (d\vec{l}_2 \times \hat{a}_{R_{21}})}{R_{21}^2}$$

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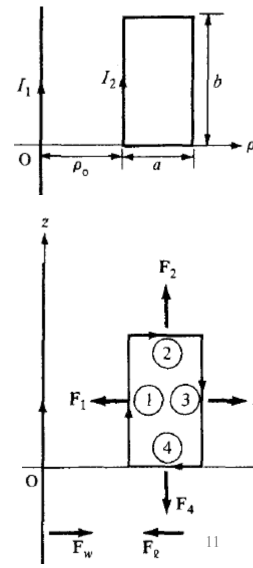
Two Current Elements

Determine the force experienced by the loop

$$\vec{F}_l = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = I_2 \oint d\vec{l}_2 \times \vec{B}_1$$



$\underline{\vec{F}_1 :}$

$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi\rho_0}\hat{a}_\phi$$

$$\vec{F}_1 = I_2 \int d\vec{l}_2 \times \vec{B}_1 = I_2 \int_{z=0}^{b} dz\,\hat{a}_z \times \frac{\mu_0 I_1}{2\pi\rho_0}\hat{a}_\phi$$

$$= -\frac{\mu_0 I_1 I_2 b}{2\pi\rho_0}\hat{a}_\rho$$

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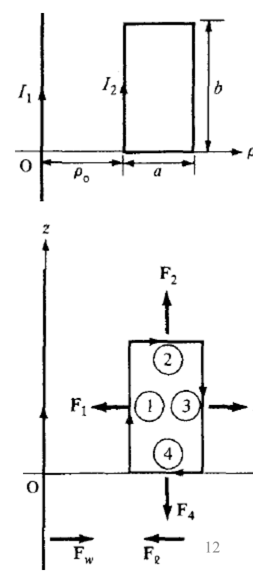
Two Current Elements

$\underline{\vec{F}_3 :}$

$$\vec{F}_3 = I_2 \int d\vec{l}_2 \times \vec{B}_1 = -I_2 \int_{z=b}^{0} dz\,\hat{a}_z \times \frac{\mu_0 I_1}{2\pi(\rho_0 + a)}\hat{a}_\phi$$

$$= \frac{\mu_0 I_1 I_2 b}{2\pi(\rho_0 + a)}\hat{a}_\rho$$



$\underline{\vec{F}_2 :}$

$$\vec{F}_2 = I_2 \int_{\rho=\rho_0}^{\rho_0 + a} d\rho\,\hat{a}_\rho \times \frac{\mu_0 I_1}{2\pi\rho}\hat{a}_\phi$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}\ln\frac{\rho_0 + a}{\rho_0}\hat{a}_z$$

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Two Current Elements

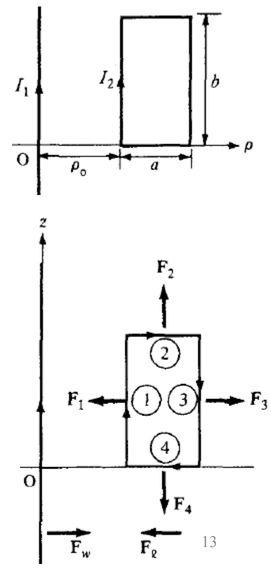
$\underline{\vec{F}_4 :}$

$$\vec{F}_4 = -I_2 \int_{\rho=\rho_0+a}^{\rho_0} d\rho\, \hat{a}_\rho \times \frac{\mu_0 I_1}{2\pi\rho} \hat{a}_\phi$$

$$= -\frac{\mu_0 I_1 I_2}{2\pi} \ln \frac{\rho_0 + a}{\rho_0} \hat{a}_z$$

$$\vec{F}_l = \frac{\mu_0 I_1 I_2}{2\pi} \left[\frac{1}{\rho_0} - \frac{1}{\rho_0 + a} \right] (-\hat{a}_\rho)$$

An attractive force trying to draw the loop toward the wire.



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Lorentz Force Law

Total force:

$$\vec{F} = \vec{F}_e + \vec{F}_m$$

$$\vec{F} = Q\left(\vec{E} + \vec{u} \times \vec{B}\right)$$

Usually, $\vec{F}_m \ll \vec{F}_e$

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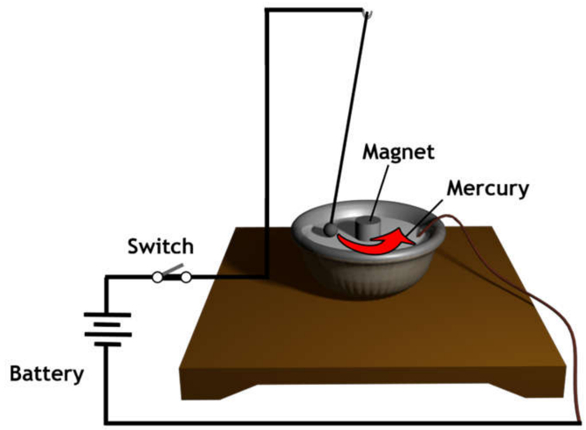
Faraday’s First Magnetic Motor



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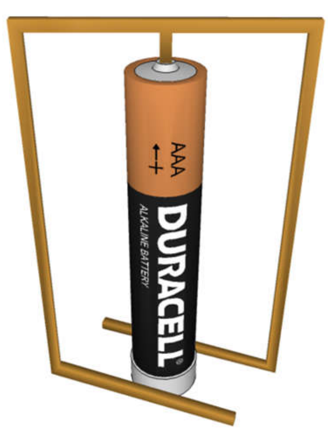
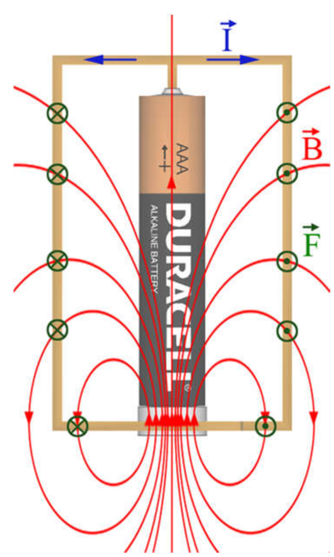
Homopolar Motor





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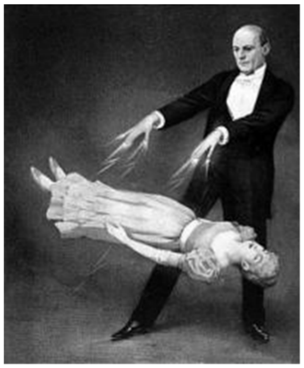
Magnetic Levitation



Shanghai Maglev Train goes up to 431 km/h

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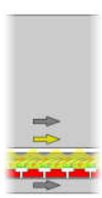
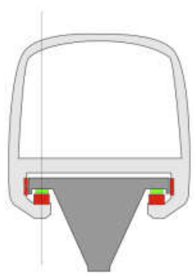
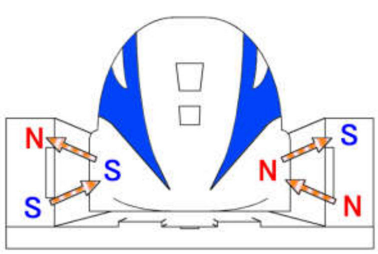
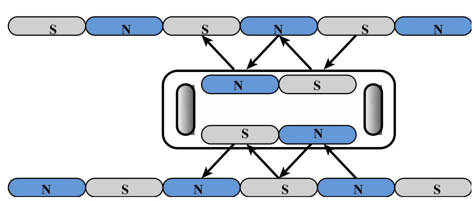
Magnetic Levitation









Maglev Propulsion

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