

# AMPERE'S CIRCUIT LAW

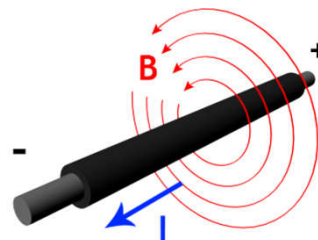
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## *Ampere's Circuit Law*

- Ampere's law is only useful for calculation in certain specific situations, involving highly symmetric currents.
- Only holds for constant fields. We will need to introduce another term when the electric field is changing with time.

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}}$$



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## *Applications of Ampere's Law*

1. Identify region in which to calculate magnetic field.
2. Choose Amperian path using symmetry: magnetic field is zero or constant of the loop.
3. Calculate  $\oint \vec{H} \cdot d\vec{l}$ .
4. Calculate current enclosed by loop  $I_{\text{enc}}$ .
5. Apply Ampere's law to solve for magnetic field

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}}$$

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## *A Toroidal Coil*

Determine  $H$  inside and outside the toroidal coil that has  $N$  turns.

**Region 1:**  $\rho < b$

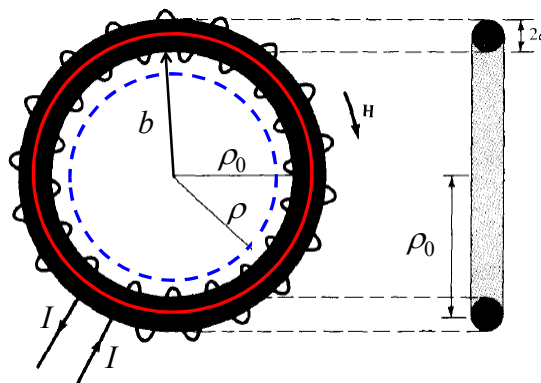
**Draw Amperian path**

Choose a concentric path with radius  $\rho$  less than  $b$ .

**Apply Ampere's Law**

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}} = 0$$

$$H = 0$$



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## A Toroidal Coil

Determine  $H$  inside and outside the toroid that has  $N$  turns.

**Region 2:**  $b < \rho < b + 2a$

**Draw Amperian path**

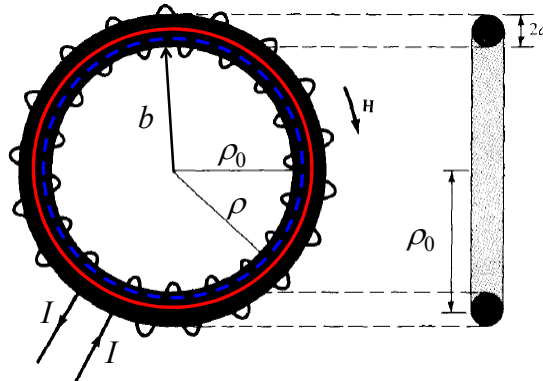
Choose a concentric path of radius  $b < \rho < b + 2a$ .

**Apply Ampere's Law**

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}}$$

$$H \cdot 2\pi\rho = NI$$

$$H = \frac{NI}{2\pi\rho} \rightarrow \vec{H} = -\frac{NI}{2\pi\rho} \hat{a}_\phi$$



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## A Toroidal Coil

Determine  $H$  inside and outside the toroid that has  $N$  turns.

**Region 3:**  $\rho > b + 2a$

**Draw Amperian path**

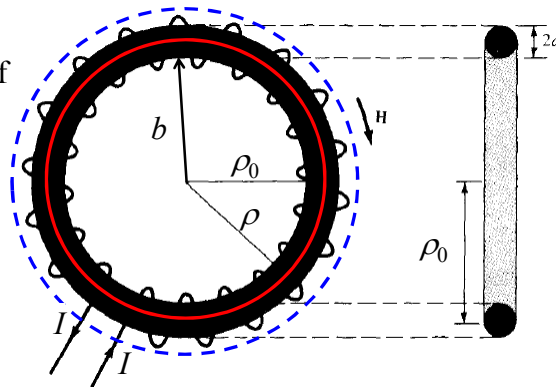
Choose a concentric path of radius  $\rho > b + 2a$ .

**Apply Ampere's Law**

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}}$$

$$H \cdot 2\pi\rho = NI - NI = 0$$

$$H = 0$$



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## Maxwell's Equation

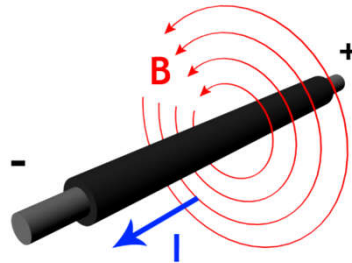
Ampere's law:

$$I_{\text{enc}} = \oint_L \vec{H} \cdot d\vec{l}$$

Using Stoke's theorem

$$\begin{aligned} I_{\text{enc}} &= \int_S (\nabla \times \vec{H}) \cdot d\vec{S} \\ &= \int_S \vec{J} \cdot d\vec{S} \end{aligned}$$

$$\nabla \times \vec{H} = \vec{J} \rightarrow \text{Third Maxwell's equation}$$



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## Magnetic Flux Density

$$\vec{B} = \mu_0 \vec{H}$$

$\mu_0$ : permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Magnetic flux through a surface  $S$

$$\psi = \int_S \vec{B} \cdot d\vec{S}$$

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