AMPERE'S CIRCUIT LAW

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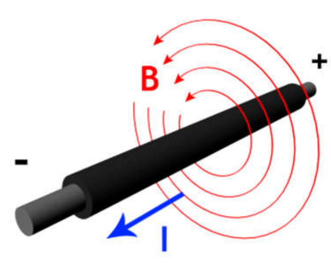
Ampere's Circuit Law

$$\oint \vec{\mathrm{H}} \cdot d\vec{\mathrm{l}} = I_{\mathrm{enc}}$$

- Ampere's law is only useful for calculation in certain specific situations, involving highly symmetric currents.
- Only holds for constant fields. We will need to introduce another term when the electric field is changing with time.



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Applications of Ampere's Law

1. Identify region in which to calculate magnetic field.
2. Choose Amperian path using symmetry: magnetic field is zero or constant of the loop.
3. Calculate $\oint \vec{H} \cdot d\vec{l}$.
4. Calculate current enclosed by loop I_{enc}.
5. Apply Ampere's law to solve for magnetic field

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

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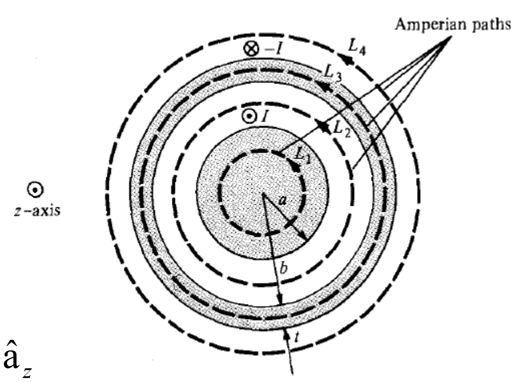
Infinitely Long Coaxial Transmission Line

Region 1: $0 \le \rho \le a$

Draw Amperian path

Choose a concentric path L_1.

Apply Ampere's Law



$$\oint_{L_1} \vec{H} \cdot d\vec{l} = I_{enc} = \int \vec{J} \cdot d\vec{S}$$

$$\vec{J} = \frac{I}{\pi a^2}\hat{a}_z, \; d\vec{S} = \rho \, d\phi \, d\rho \, \hat{a}_z$$

$$I = \int \vec{J} \cdot d\vec{S} = \frac{I}{\pi a^2}\iint \rho \, d\phi \, d\rho = \frac{I}{\pi a^2}\pi\rho^2 = \frac{I\rho^2}{a^2}$$

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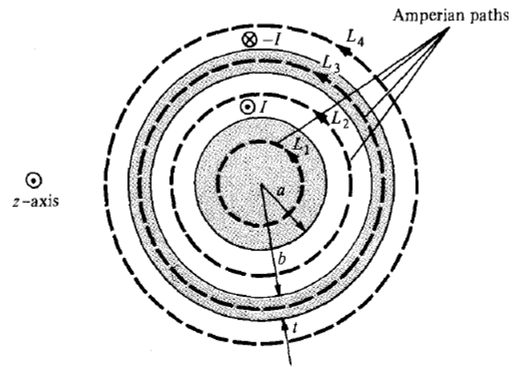
Infinitely Long Coaxial Transmission Line

Region 1: $0 \le \rho \le a$

$$H_\phi \int dl = H_\phi 2\pi\rho = \frac{I\rho^2}{a^2}$$

$$H_\phi = \frac{I\rho}{2\pi a^2}$$



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Infinitely Long Coaxial Transmission Line

Region 2: $a \le \rho \le b$

Draw Amperian path

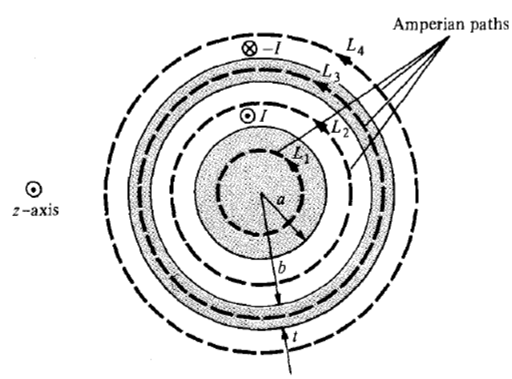
Choose a concentric path L_2.

Apply Ampere's Law

$$\oint_{L_2} \vec{H} \cdot d\vec{l} = I$$

$$H_\phi 2\pi\rho = I$$

$$H_\phi = \frac{I}{2\pi\rho}$$



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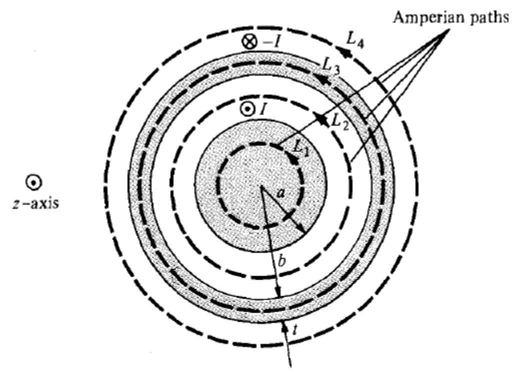
Infinitely Long Coaxial Transmission Line

Region 3: $b \le \rho \le b + t$

Draw Amperian path

Choose a concentric path L_3.

Apply Ampere's Law



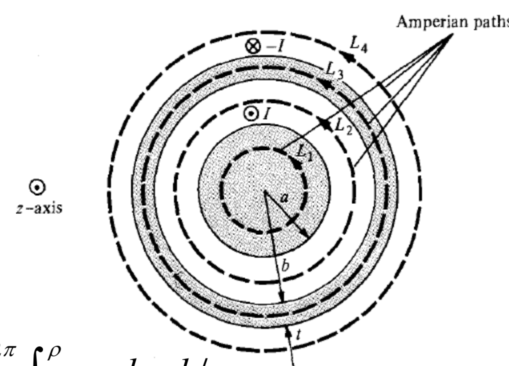
$$\oint_{L_3} \vec{H} \cdot d\vec{l} = I_{\text{enc}}$$

$$H_\phi 2\pi\rho = I_{\text{enc}}$$

$$I_{\text{enc}} = I + \int \vec{J} \cdot d\vec{S}$$

Infinitely Long Coaxial Transmission Line

Region 3: $b \le \rho \le b + t$



$$\vec{J} = -\frac{I}{\pi\left[(b+t)^2 - b^2\right]}\hat{a}_z$$

$$I_{\text{enc}} = I - \frac{I}{\pi\left[(b+t)^2 - b^2\right]}\int_{\phi=0}^{2\pi}\int_{\rho=b}^{\rho} \rho\, d\rho\, d\phi$$

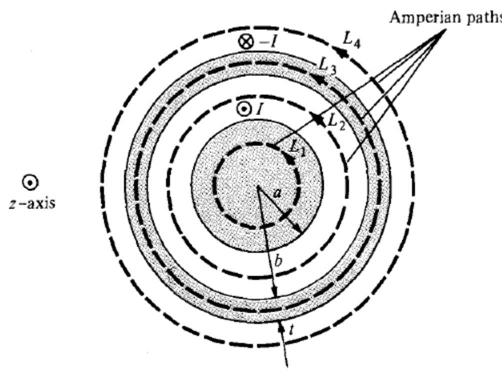
$$= I\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$

Infinitely Long Coaxial Transmission Line

Region 3: $b \le \rho \le b+t$

$$H_\phi 2\pi\rho = I_{\text{enc}}$$

$$H_\phi = \frac{I}{2\pi\rho}\left[1 - \frac{\rho^2 - b^2}{t^2 + 2bt}\right]$$



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Infinitely Long Coaxial Transmission Line

Region 4: $\rho \ge b+t$

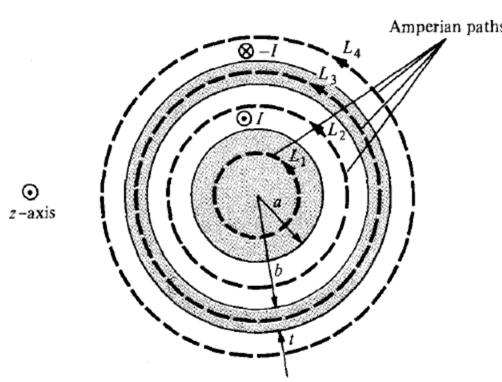
Draw Amperian path

Choose a concentric path L_4.

Apply Ampere's Law

$$\oint_{L_4} \vec{H} \cdot d\vec{l} = I_{\text{enc}} = I - I = 0$$

$$H_\phi = 0$$



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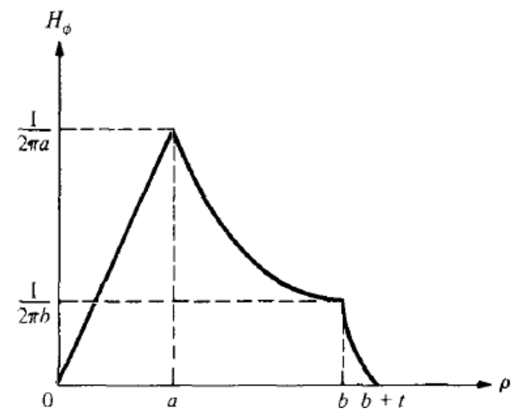
Infinitely Long Coaxial Transmission Line



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