# VECTOR ALGEBRA

## *First Assignment - Reminder*

- If you already have not done this!
- Email to
  anis@eee.buet.ac.bd with
  - **Subject:** EEE 209
  - **Body:** "Your Name, Student Number" <email address>
- It will help me to keep you posted on the course updates.

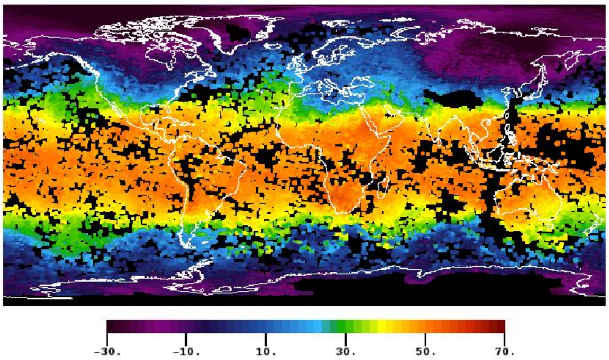
2

# *Scalars*

A **scalar** quantity has only magnitude
– time, mass, distance, temperature, potential.

***Representation:*** $A, B \rightarrow$ scalars

**Global night time air temperature**



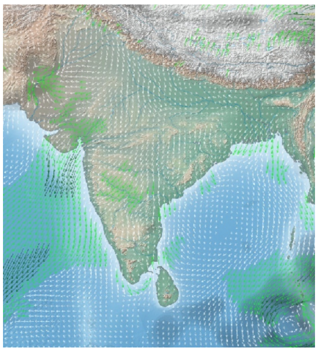

3

# *Vectors*

A **vector** quantity has both magnitude and direction
– velocity, force, electric field.

***Representation:*** $\vec{A}, \vec{B} \rightarrow$ vectors

**Wind flow chart**

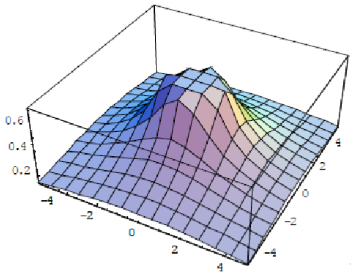


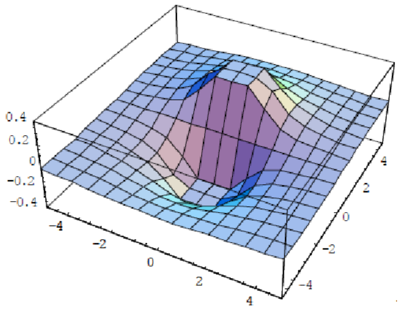
4

# *Fields*

A **field** is a function that specifies a particular quantity everywhere in a region.

**Scalar field** – 1. temperature distribution in a building
2. electric potential in a region



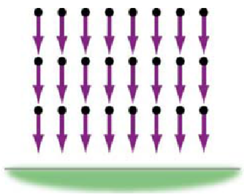



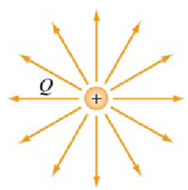

- Electric potential due to a point charge located at the origin.
- Potential due to a dipole with the positive charge located at $y = 1$ and the negative charge at $y = -1$.

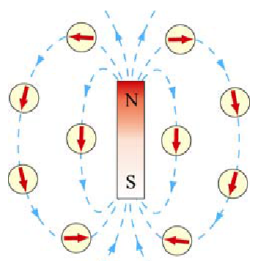


# *Fields*

**Vector field** – 1. gravitational force on a body in space
2. velocity of raindrops in the atmosphere






- Gravitational field
- Electric field for positive charge
- Magnetic field of a bar magnet

# Unit Vector

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|}$$

$$|\hat{a}_A| = 1$$

$$\vec{A} = A\hat{a}_A$$

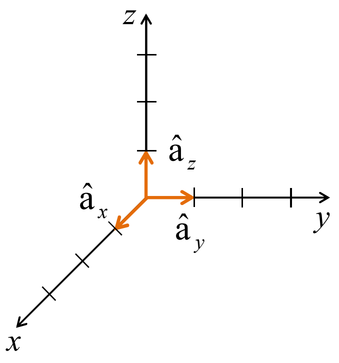


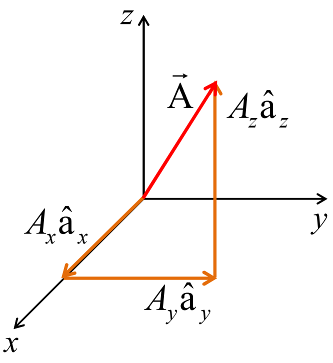
# Unit Vectors and Components

In Cartesian coordinates

$$\vec{A} = (A_x, A_y, A_z) \text{ or } A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z$$

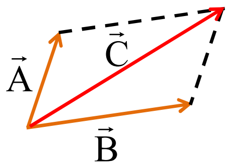
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}, \ \hat{a}_A = \frac{A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$



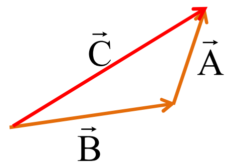



# Vector Addition

$$\vec{C} = \vec{A} + \vec{B} = \vec{B} + \vec{A}$$




(a) Parallelogram rule




(b) Head-to-tail rule



# Equality of Two Vectors

$$\vec{A} = \hat{a}_A A = \hat{a}_x A_x + \hat{a}_y A_y + \hat{a}_z A_z$$

$$\vec{B} = \hat{a}_B B = \hat{a}_x B_x + \hat{a}_y B_y + \hat{a}_z B_z$$

- Conditions for $\vec{A} = \vec{B}$ : $A = B$

$$\hat{a}_A = \hat{a}_B$$

$$A_x = B_x$$

$$A_y = B_y$$

$$A_z = B_z$$

# Vector Decomposition: 2D

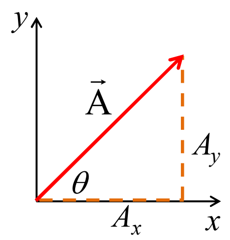
Consider a vector: $\vec{A} = (A_x, A_y, 0)$

Components: $A_x = A\cos\theta,\ A_y = A\sin\theta$

**Magnitude:** $A = \sqrt{A_x^2 + A_y^2}$

**Direction:** $\frac{A_y}{A_x} = \frac{A\sin(\theta)}{A\cos(\theta)} = \tan(\theta)$

$$\theta = \tan^{-1}\frac{A_y}{A_x}$$




11

# Vector Multiplication

**Simple Product:**

$$\vec{B} = k\vec{A}$$
$$= \hat{a}_A kA$$
$$= \hat{a}_x(kA_x) + \hat{a}_y(kA_y) + \hat{a}_z(kA_z)$$

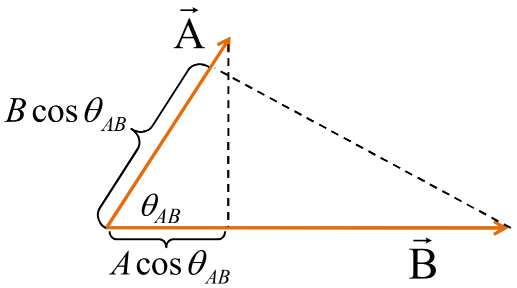
# *Vector Multiplication*

**Scalar or Dot Product:**

$$\vec{A}\cdot\vec{B} = AB\cos\theta_{AB} = C$$

$$\vec{A}\cdot\vec{B} = A\left(B\cos\theta_{AB}\right)$$

$$\vec{A}\cdot\vec{B} = B\left(A\cos\theta_{AB}\right)$$




# *Vector Multiplication*

**Scalar or Dot Product:**

Note that

$$\hat{a}_x\cdot\hat{a}_y = \hat{a}_y\cdot\hat{a}_z = \hat{a}_z\cdot\hat{a}_x = 0$$

$$\hat{a}_x\cdot\hat{a}_x = \hat{a}_y\cdot\hat{a}_y = \hat{a}_z\cdot\hat{a}_z = 1$$

If $\vec{A} = \left(A_x, A_y, A_z\right)$ And $\vec{B} = \left(B_x, B_y, B_z\right)$, then

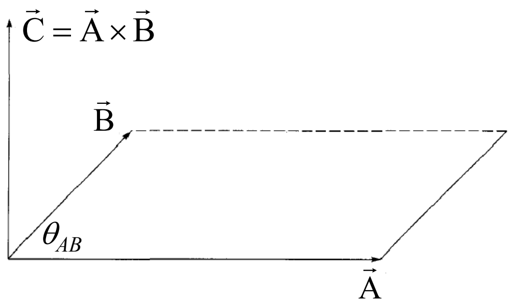
$$\vec{A}\cdot\vec{B} = \left(\hat{a}_x A_x + \hat{a}_y A_y + \hat{a}_z A_z\right)\cdot\left(\hat{a}_x B_x + \hat{a}_y B_y + \hat{a}_z B_z\right)$$
$$= A_x B_x + A_y B_y + A_z B_z$$

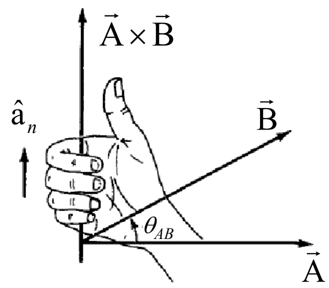
# Vector Multiplication

**Vector or Cross Product:**

$$\vec{A} \times \vec{B} = AB \sin\theta_{AB} \hat{a}_n = \vec{C}$$

$\hat{a}_n$ : normal to the plane containing $\vec{A}$ and $\vec{B}$.



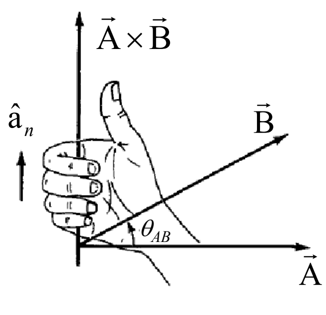




# Vector Multiplication

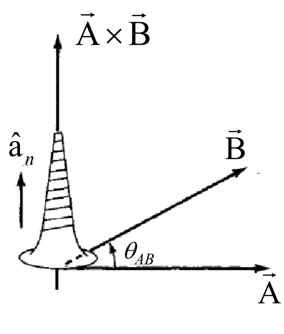
**Vector or Cross Product:**

Direction of $\hat{a}_n$:

Right-hand rule




Right-handed screw rule



# *Vector Multiplication*

**Vector or Cross Product:**

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{a}_x & \hat{a}_y & \hat{a}_z\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$=(A_yB_z-A_zB_y)\hat{a}_x+(A_zB_x-A_xB_z)\hat{a}_y+(A_xB_y-A_yB_x)\hat{a}_z$$

# *Vector Multiplication*

**Vector (or Cross) Product:**

It is not commutative $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$

It is anti-commutative $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$

It is not associative $\vec{A}\times(\vec{B}\times\vec{C})\neq(\vec{A}\times\vec{B})\times\vec{C}$

It is distributive $\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$

$$\vec{A}\times\vec{A}=0$$

$$\hat{a}_x\times\hat{a}_y=\hat{a}_z$$

$$\hat{a}_y\times\hat{a}_z=\hat{a}_x$$

$$\hat{a}_z\times\hat{a}_x=\hat{a}_y$$