MAGNETOSTATICS

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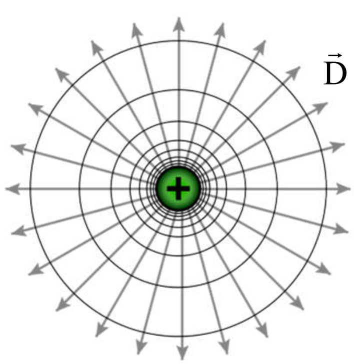
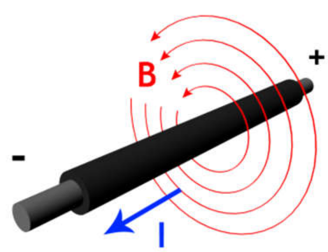
Magnestatic Fields





$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

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Major Laws

Electrostatics	Magnetostatics
Coulomb's law	Bio-Savart's law
Gauss's law	Ampere's Circuit law

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Biot-Savart's Law

dH at a point *P* by the differential current element *I dl* is

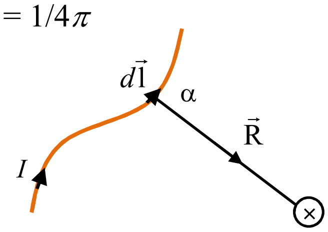
1. proportional to the product *I dl*
2. proportional to the sine of the angle α between *dl* and *R*
3. inversely proportional to the square of *R*.

$$dH = \frac{kIdl\sin\alpha}{R^2}$$

In SI units, $k = 1/4\pi$

$$dH = \frac{Idl\sin\alpha}{4\pi R^2}$$

$$d\vec{H} = \frac{I\,d\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{I\,d\vec{l} \times \vec{R}}{4\pi R^3}$$



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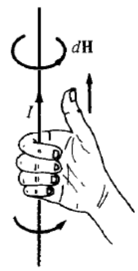
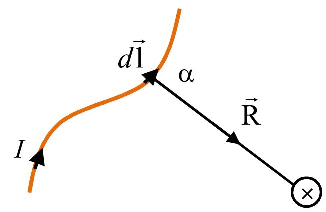
Direction of dH



Right-hand rule

Right-handed screw rule



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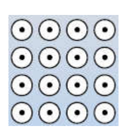
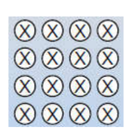
Notation of Direction of dH



$\rightarrow$ Arrow Head

out of the page



$\rightarrow$ Arrow Tail

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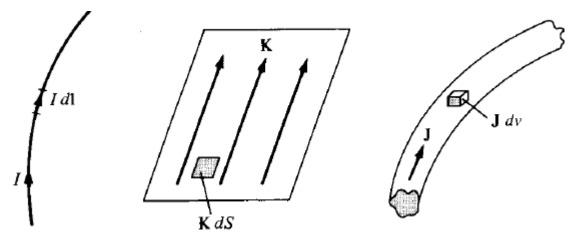
Charge Configurations

$$\vec{H} = \int_L \frac{I\, d\vec{l} \times \hat{a}_R}{4\pi R^2} \qquad \text{(line current)}$$

$$\vec{H} = \int_S \frac{\vec{K}\, dS \times \hat{a}_R}{4\pi R^2} \qquad \text{(surface current)}$$

$$\vec{H} = \int_v \frac{\vec{J}\, dv \times \hat{a}_R}{4\pi R^2} \qquad \text{(volume current)}$$



line current surface current volume current

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Line Current

Determine $\vec{H}$ at P

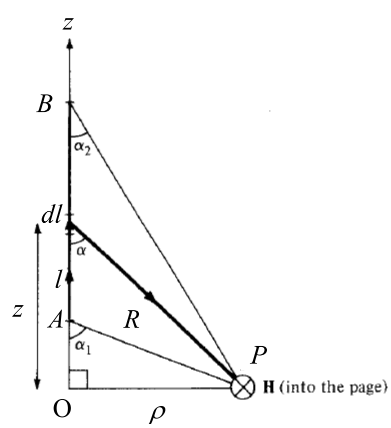
Use Biot-Savart's law

$$d\vec{H} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$

$$d\vec{l} = dz\,\hat{a}_z \quad \vec{R} = \rho\,\hat{a}_\rho - z\,\hat{a}_z$$

$$d\vec{l} \times \vec{R} = \rho\, dz\,\hat{a}_\phi$$

$$\vec{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2 + z^2]^{3/2}}\hat{a}_\phi$$



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Line Current

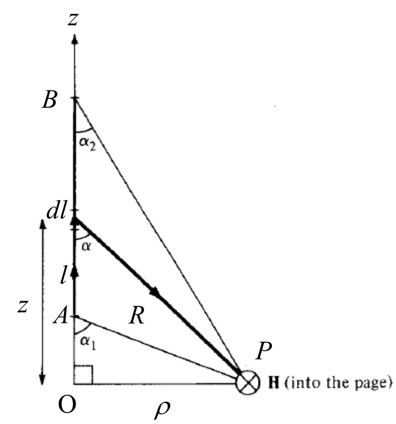
$$\vec{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2 + z^2]^{3/2}}\hat{a}_\phi$$

$$z = \rho \cot\alpha,\ dz = -\rho\, \mathrm{cosec}^2\alpha\, d\alpha$$

$$\vec{H} = -\frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2\, \mathrm{cosec}^2\alpha\, d\alpha}{\rho^3\, \mathrm{cosec}^3\alpha}\hat{a}_\phi$$

$$= -\frac{I}{4\pi\rho}\hat{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha$$

$$= \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$$



For infinite conductor, $\vec{H} = \dfrac{I}{2\pi\rho}\hat{a}_\phi$

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Circular Loop

Use Biot-Savart's law

$$d\vec{H} = \frac{I\, d\vec{l} \times \vec{R}}{4\pi R^3}$$

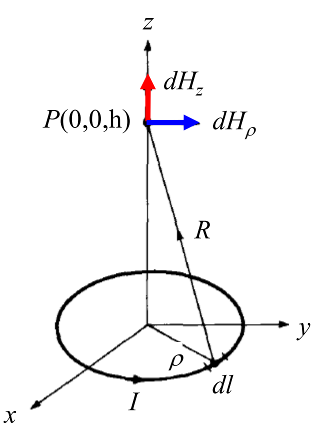
$$d\vec{l} = \rho\, d\phi\, \hat{a}_\phi$$

$$\vec{R} = (0,0,h) - (x,y,0) = -\rho\,\hat{a}_\rho + h\,\hat{a}_z$$

$$d\vec{l} \times \vec{R} = \rho h\, d\phi\, \hat{a}_\rho + \rho^2\, d\phi\, \hat{a}_z$$

$$d\vec{H} = \frac{I}{4\pi\left[\rho^2 + h^2\right]^{3/2}}\left(\rho h\, d\phi\, \hat{a}_\rho + \rho^2\, d\phi\, \hat{a}_z\right)$$

$$= dH_\rho\, \hat{a}_\rho + dH_z\, \hat{a}_z$$



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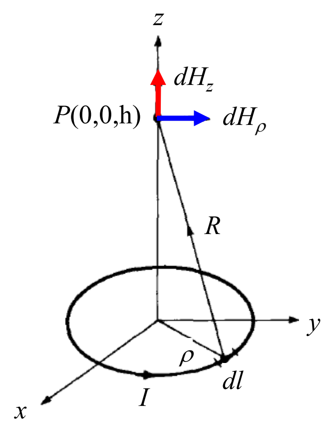
Circular Loop

Apply symmetry:

$$\vec{H}_\rho = \int dH_\rho \, \hat{a}_\rho = 0$$

$$\vec{H} = \int dH_z \, \hat{a}_z = \int_0^{2\pi} \frac{I \rho^2 \, d\phi \, a_z}{4\pi \left[\rho^2 + h^2 \right]^{3/2}}$$

$$= \frac{I \rho^2 \, \hat{a}_z}{2 \left[\rho^2 + h^2 \right]^{3/2}}$$



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