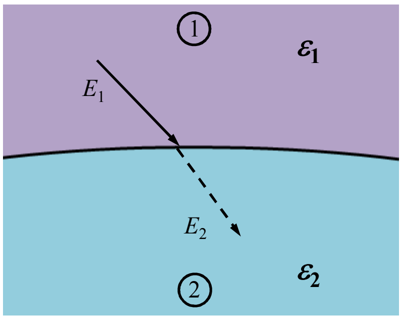
BOUNDARY CONDITIONS

Two Media

Two different media:

1. Dielectric and dielectric
2. Conductor and dielectric
3. Conductor and free space



Two Media

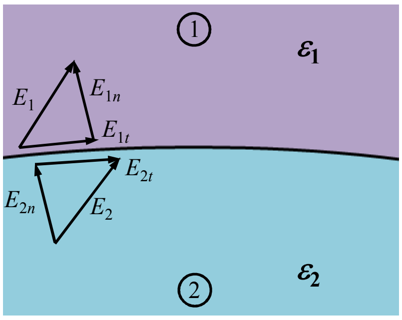
$$\vec{E}_2 = ?$$

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$$

$$\vec{E}_2 = \vec{E}_{2t} + \vec{E}_{2n}$$

$$\vec{E}_{2t} = ?$$

$$\vec{E}_{2n} = ?$$

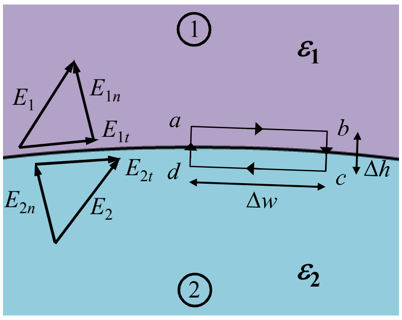


- **Finding Tangential Components:** Determine $\oint_{abcda} \vec{E}\cdot d\vec{l} = 0$
- **Finding Normal Components:** Apply Gauss's Law

Dielectric-Dielectric

$$\vec{E}_{2t} = ?$$

Determine $\oint_{abcda} \vec{E}\cdot d\vec{l} = 0$



$$E_{1t}\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w$$

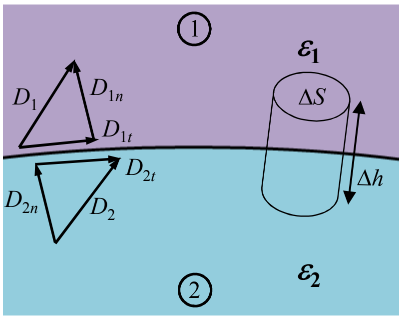
$$+E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} = 0$$

As $\Delta h \to 0$: $\boxed{E_{1t} = E_{2t}}$ $\boxed{\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}}$

***E_t* is continuous, but *D_t* is discontinuous across the boundary!**

Dielectric-Dielectric

$$\vec{E}_{2n} = ?$$



Apply Gauss's Law

$$\Delta Q = \rho_S \Delta S = D_{1n}\Delta S - D_{2n}\Delta S$$

ρ_S: Free charge density at the boundary

$$\boxed{D_{1n} - D_{2n} = \rho_S}$$

If no free charge at the interface: $\boxed{D_{1n} = D_{2n}}$ $\boxed{\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}}$

D_n is continuous, but E_n is discontinuous across the boundary!

5

Law of Refraction

- Tangential components:

$$E_{1t} = E_{2t}$$

$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

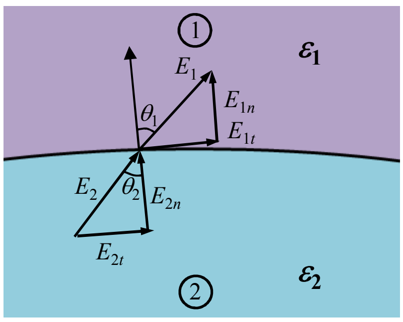
- Normal components:

$$D_{1n} = D_{2n}$$

$$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$$

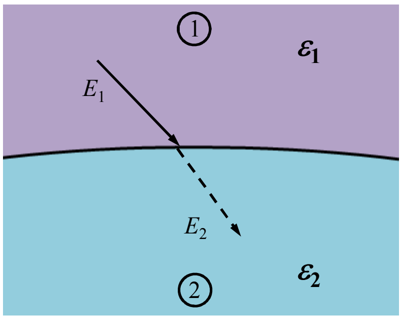
- Dividing first by second:

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$



6

Conductor-Dielectric



What are the boundary conditions for a metal-dielectric interface?

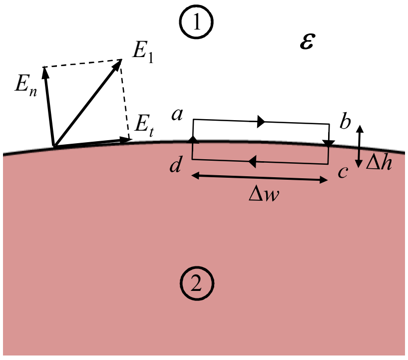
Conductor-Dielectric

$$\vec{E}_{2t} = ?$$

Determine $\oint_{abcda} \vec{E} \cdot d\vec{l} = 0$

$$E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2} - 0 \cdot \Delta w$$
$$+0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} = 0$$

As $\Delta h \to 0$, $\boxed{E_t = 0}$ $\boxed{D_t = 0}$



Conductor-Dielectric

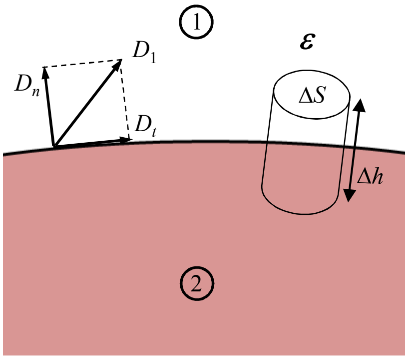
$\vec{E}_{2n} = ?$

Apply Gauss's Law

$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S$

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

$$E_n = \frac{\rho_S}{\varepsilon}$$



The electric field can be external to the conductor and normal to its surface!

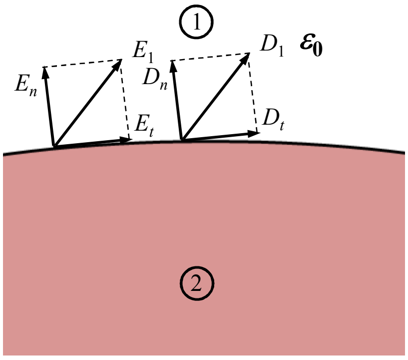
Conductor-Free Space

Special case of Conductor-Dielectric interface.

$$E_t = 0, D_t = 0$$

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S$$

$$E_n = \frac{\rho_S}{\varepsilon_0}$$



Example

Two extensive homogeneous isotropic dielectrics meet on plane $z = 0$. For $z \geq 0$, $\varepsilon_{r1} = 4$ and for $z \leq 0$, $\varepsilon_{r2} = 3$. A uniform electric field $\vec{E}_1 = 5\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z$ kV/m exists for $z \geq 0$. Find

(a) $\vec{E}_2$ for $z \leq 0$

(b) The angles E_1 and E_2 make with the interface

(c) The energy densities in Joules/m^3 in both dielectrics

(d) The energy within a cube of side 2 m centered at (3, 4, −5).