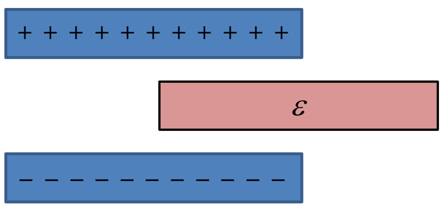
CAPACITORS

Concept Question: Dielectric

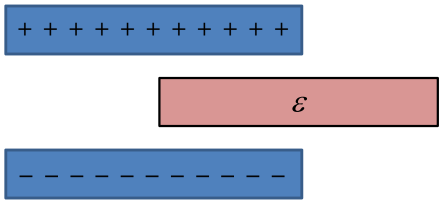
A parallel plate capacitor is charged to a total charge Q and the battery removed. A slab of material with dielectric constant ε is inserted between the plates. The **charge** stored in the capacitor



1. increases
2. decreases
3. remains the same.

Concept Question: Dielectric

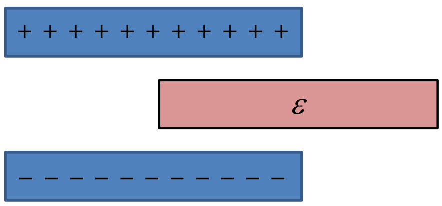
A parallel plate capacitor is charged to a total charge Q and the battery removed. A slab of material with dielectric constant ε is inserted between the plates. The **energy** stored in the capacitor



1. increases
2. decreases
3. remains the same.

Concept Question: Dielectric

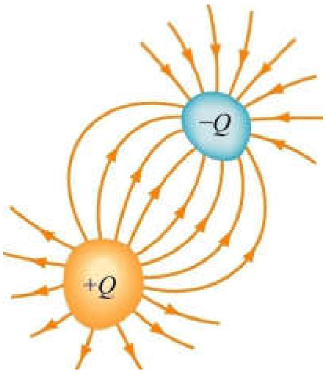
A parallel plate capacitor is charged to a total charge Q and the battery removed. A slab of material with dielectric constant ε is inserted between the plates. The **force** on the dielectric



1. pulls in the dielectric
2. pushes out the dielectric
3. is zero.

Capacitors: Store Electric Charge

Capacitor: Two isolated conductors
Equal and opposite charges $\pm Q$
Potential difference V between them.



$$C = \frac{Q}{V}$$

Units: Coulombs/Volt or Farads
C is always positive.

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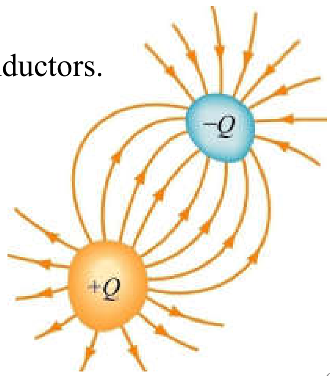
Finding Capacitance

1. Assume Q and determine V in terms of Q
2. Assume V and determine Q in terms of V

We will discuss the former method:
1. Choose a suitable coordinate system.
2. Assume $+Q$ and $-Q$ charges in the conductors.
3. Determine E and V.
4. Find C.



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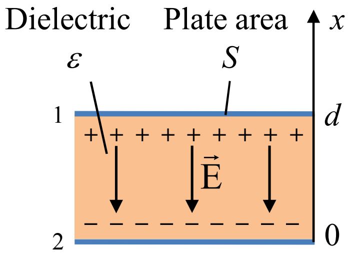
Parallel-Plate Capacitor

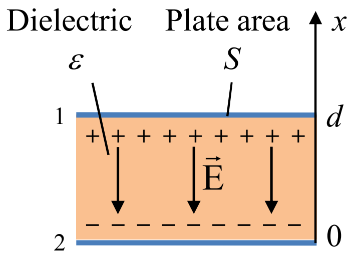


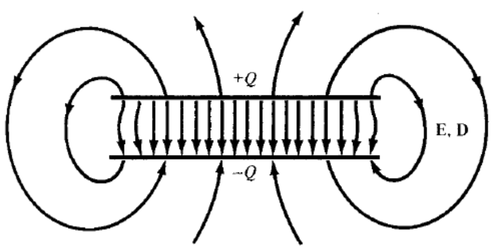
Plates 1 and 2 carry charges $+Q$ and $-Q$ uniformly distributed on them.

Parallel-Plate Capacitor

Consider homogeneous dielectric between the plates.

Neglect fringing at the edges.





Parallel-Plate Capacitor

Determine *E*

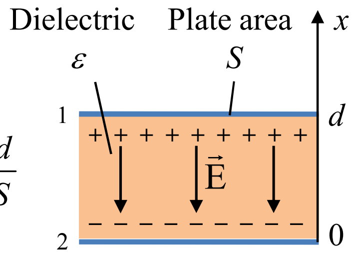
$$\rho_S = \frac{Q}{S} \rightarrow \vec{E} = \frac{\rho_S}{\varepsilon}(-\hat{a}_x) = -\frac{Q}{\varepsilon S}\hat{a}_x$$

Determine *V*

$$V = -\int_2^1 \vec{E}\cdot d\vec{l} = -\int_0^d \left[-\frac{Q}{\varepsilon S}\hat{a}_x\right]\cdot dx\,\hat{a}_x = \frac{Qd}{\varepsilon S}$$

Determine *C*

$$C = \frac{Q}{V} = \frac{\varepsilon S}{d}$$



Dielectric constant ε_r can be measured by measuring *C*!

$$\varepsilon_r = \frac{C}{C_0}$$

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Energy Stored in Capacitor

Start Charging Capacitor

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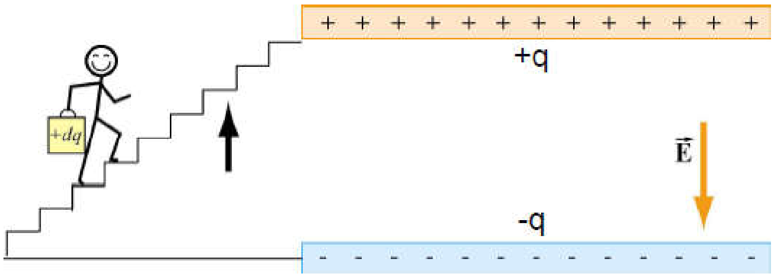
Energy to Charge Capacitor



1. Capacitor starts uncharged.
2. Carry $+dq$ from bottom to top. Now top has charge $q = +dq$, bottom $-dq$.
3. Repeat.
4. Finish when top has charge $q = +Q$, bottom $-Q$.

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Work Done Charging Capacitor

Potential difference is: $V = q / C$

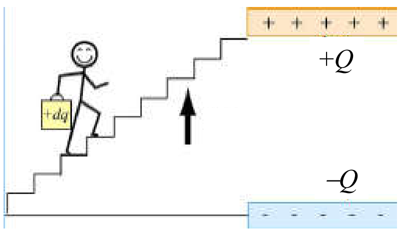
So work done to move dq is:

$$dW = dq\,V = dq\frac{q}{C} = \frac{1}{C} q\,dq$$

Total energy to charge to $q = Q$:

$$W = \int dW = \frac{1}{C}\int_0^Q q\,dq$$

$$W = \frac{Q^2}{2C}$$



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Work Done Charging Capacitor

Since, $C = \frac{Q}{V}$

$$W = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Where is the energy stored?

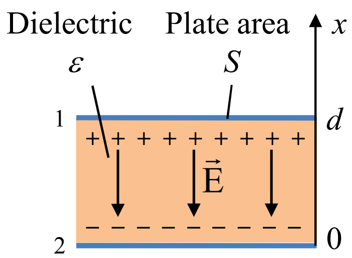
Energy Stored in Capacitor

Energy stored in the E field!

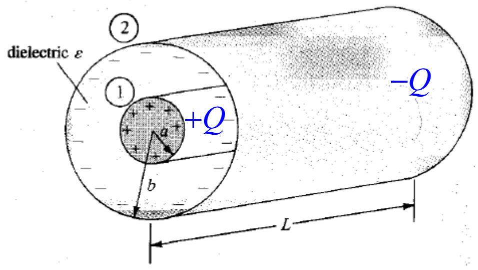
$$W_E = \frac{1}{2}\int \varepsilon E^2\, dv$$

$$= \frac{1}{2}\int \varepsilon \frac{Q^2}{\varepsilon^2 S^2}\, dv = \frac{1}{2}\frac{\varepsilon Q^2 Sd}{\varepsilon^2 S^2}$$

$$= \frac{Q^2}{2}\left(\frac{d}{\varepsilon S}\right) = \frac{Q^2}{2C} = \frac{1}{2}QV$$



Coaxial Capacitor



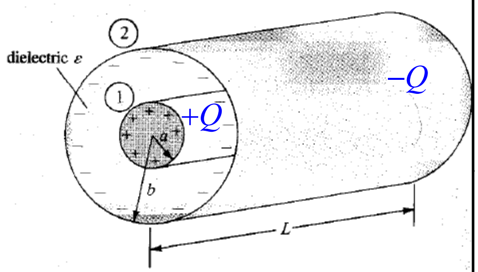
- Conductors 1 and 2 carry charges $+Q$ and $-Q$ uniformly distributed on them.
- Consider homogeneous dielectric between the plates.
- Neglect fringing at the edges.

Coaxial Capacitor

Determine *E*

$$Q = \varepsilon \oint \vec{E} \cdot d\vec{S} = \varepsilon E_\rho 2\pi\rho L$$

$$\vec{E} = \frac{Q}{2\pi\varepsilon\rho L}\hat{a}_\rho$$



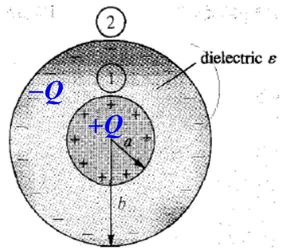
Determine *V*

$$V = -\int_2^1 \vec{E} \cdot d\vec{l} = -\int_b^a \left[\frac{Q}{2\pi\varepsilon\rho L}\hat{a}_\rho\right] \cdot d\rho\,\hat{a}_\rho$$

$$= \frac{Q}{2\pi\varepsilon L}\ln\frac{b}{a}$$

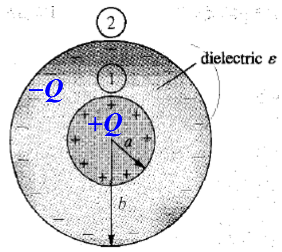
Determine *C* $\quad C = \dfrac{Q}{V} = \dfrac{2\pi\varepsilon L}{\ln\dfrac{b}{a}}$

Spherical Capacitor



Inner and outer spheres carry charges +Q and –Q uniformly distributed on them.

Spherical Capacitor



Determine *E*

$$Q = \varepsilon \oint \vec{E} \cdot d\vec{S} = \varepsilon E_r 4\pi r^2$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{a}_r$$

Determine *V*

$$V = -\int_2^1 \vec{E} \cdot d\vec{l} = -\int_b^a \left[\frac{Q}{4\pi\varepsilon r^2}\hat{a}_r\right] \cdot dr\,\hat{a}_r$$

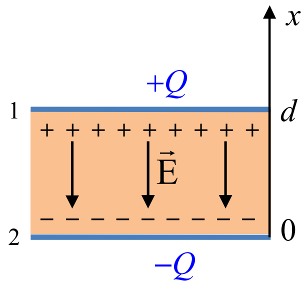
$$= \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Determine *C* $\quad C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{b}\right]}$

Concept Question: Changing Dimensions

A parallel plate capacitor has plates with equal and opposite charges ±*Q*, separated by a distance *d*, and is not connected to a battery. The plates are pulled apart to a distance *D*>*d*. What happens?

1. V increases, Q increases
2. V decreases, Q increases
3. V is the same, Q increases
4. V increases, Q is the same
5. V decreases, Q is the same
6. V is the same, Q is the same
7. V increases, Q decreases
8. V decreases, Q decreases
9. V is the same, Q decreases



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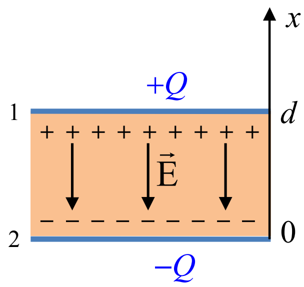
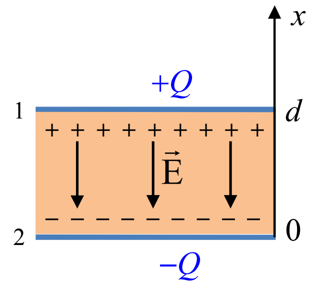


Concept Question: Changing Dimensions

A parallel plate capacitor has plates with equal and opposite charges $\pm Q$, separated by a distance d, and is not connected to a battery. The plates are pulled apart to a distance $D>d$. How does the final electrostatic energy stored in the capacitor compare to the initial energy?

1. The final stored energy is smaller
2. The final stored energy is large
3. Stored energy does not change



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