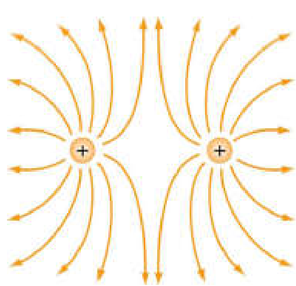
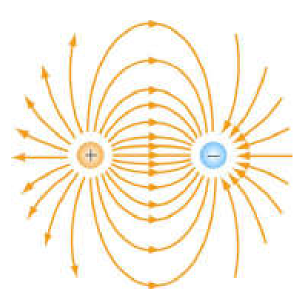
FIELD LINES AND EQUIPOTENTIAL

Electric Field Lines

- Direction of field at any point is tangent to field line at that point.
- Field lines point away from positive charges and terminate on negative charges.
- Field lines never cross each other.



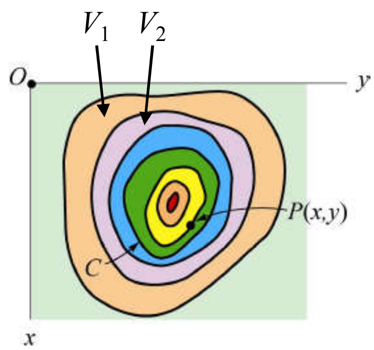


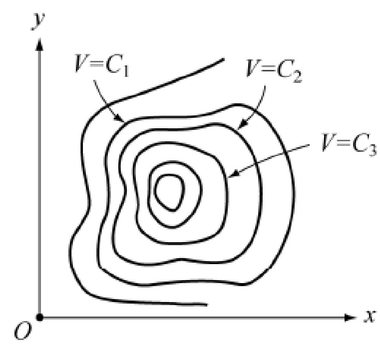
Equipotential

Equipotential surface: Potential is same throughout the surface.

Equipotential line: Potential is same along the line

$$\int \vec{E} \cdot d\vec{l} = 0 \text{ on an equipotential surface or line}$$





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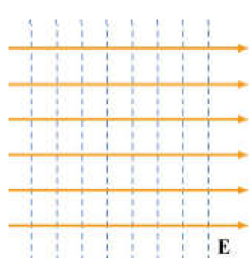
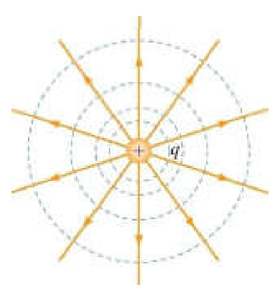
Direction of Electric Field

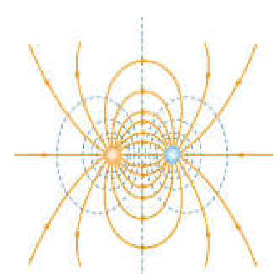
Electric field is perpendicular to all equipotentials.



Constant *E* Field



Point Charge



Electric Dipole

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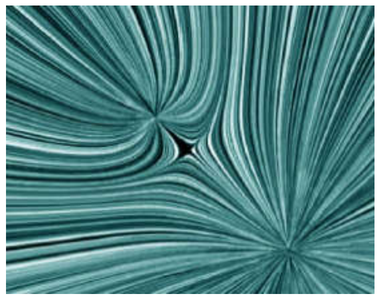
Concept Question: Force

Field lines around two charges



The force between the two charges is:

1. Attractive
2. Repulsive
3. Need more information.

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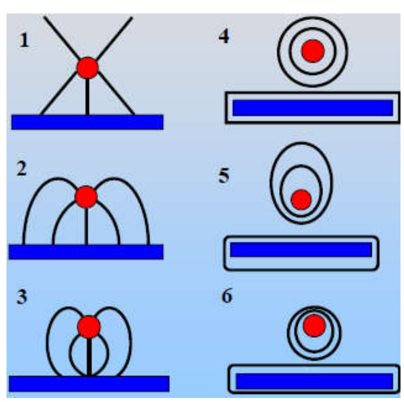
Concept Question: Equipotential

The circle is at +5 V relative to the plate. Which of the below is the most accurate equipotential map?



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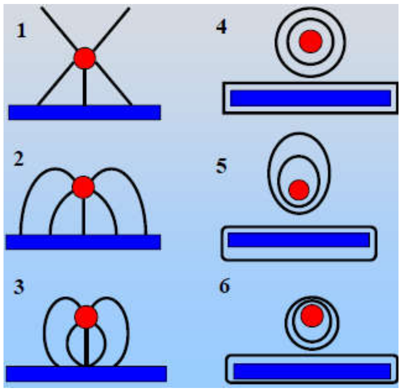
Concept Question: Field Lines

The circle is at +5 V relative to the plate. Which of the below is the most accurate electric field line map?



ENERGY DENSITY

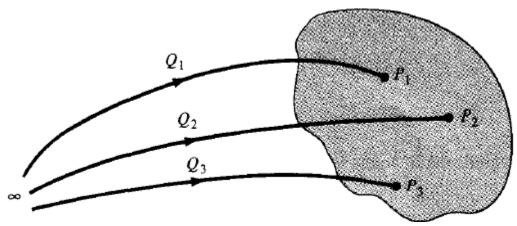
Energy Density

Total work done in positioning Q_1, Q_2, and Q_3 in order Q_1, Q_2, and Q_3

$$\begin{aligned} W_E &= W_1 + W_2 + W_3 \\ &= 0 + Q_2 V_{21} + Q_3 \left(V_{31} + V_{32} \right) \end{aligned}$$

Total work done in positioning Q_1, Q_2, and Q_3 in order Q_3, Q_2, and Q_1

$$\begin{aligned} W_E &= W_3 + W_2 + W_1 \\ &= 0 + Q_2 V_{23} + Q_1 \left(V_{12} + V_{13} \right) \end{aligned}$$



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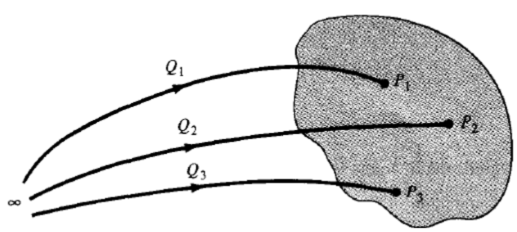
Energy Density

Adding work done to transfer charges in both orders

$$\begin{aligned} 2W_E &= Q_1 \left(V_{12} + V_{13} \right) + Q_2 \left(V_{21} + V_{23} \right) + Q_3 \left(V_{31} + V_{32} \right) \\ &= Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \end{aligned}$$

$$W_E = \frac{1}{2} \left(Q_1 V_1 + Q_2 V_2 + Q_3 V_3 \right)$$

For n point charges → $\boxed{W_E = \frac{1}{2} \sum_{k=1}^{n} Q_k V_k}$



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Energy Density

For continuous charge distribution

$$W_E = \frac{1}{2}\int_L \rho_L V\, dl \qquad \text{(Line charge)}$$

$$W_E = \frac{1}{2}\int_S \rho_S V\, dS \qquad \text{(Surface charge)}$$

$$W_E = \frac{1}{2}\int_v \rho_v V\, dv \qquad \text{(Volume charge)}$$

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Energy Density

For continuous charge distribution

$$W_E = \frac{1}{2}\int_v \rho_v V\, dv = \frac{1}{2}\int_v \left(\nabla\cdot\vec{\mathrm{D}}\right) V\, dv$$

From Vector Calculus → $\nabla\cdot V\vec{\mathrm{A}} = \vec{\mathrm{A}}\cdot\nabla V + V\left(\nabla\cdot\vec{\mathrm{A}}\right)$

$$\left(\nabla\cdot\vec{\mathrm{A}}\right)V = \nabla\cdot V\vec{\mathrm{A}} - \vec{\mathrm{A}}\cdot\nabla V$$

After substitution → $W_E = \frac{1}{2}\int_v \left(\nabla\cdot V\vec{\mathrm{D}}\right)dv - \frac{1}{2}\int_v \left(\vec{\mathrm{D}}\cdot\nabla V\right)dv$

Using Divergence theorem → $W_E = \frac{1}{2}\oint_S \left(V\vec{\mathrm{D}}\right)\cdot d\vec{\mathrm{S}} - \frac{1}{2}\int_v \left(\vec{\mathrm{D}}\cdot\nabla V\right)dv$

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Energy Density

$$W_E = \frac{1}{2}\oint_S \left(V\vec{D}\right)\cdot d\vec{S} - \frac{1}{2}\int_v \left(\vec{D}\cdot\nabla V\right)dv$$

For point charges →

- V falls off as $1/r$
- $\vec{D}$ falls off as $1/r^2$

For dipoles →

- V falls off as $1/r^2$
- $\vec{D}$ falls off $1/r^3$

We can neglect the first term in the above equation!

$$W_E = -\frac{1}{2}\int_v \left(\vec{D}\cdot\nabla V\right)dv = \frac{1}{2}\int_v \left(\vec{D}\cdot\vec{E}\right)dv = \frac{1}{2}\int_v \varepsilon_0 E^2 dv$$

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