Self-induced transparency modelocking of quantum cascade lasers in the presence of saturable nonlinearity and group velocity dispersion

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Abstract: We consider the impact of saturable nonlinearity and group velocity dispersion on self-induced transparency (SIT) modelocking of quantum cascade lasers (QCLs). We find that self-induced transparency modelocking in QCLs can be obtained in the presence of saturable nonlinearity if the saturable loss or gain is below a critical limit. The limit for the saturable loss is significantly more stringent than the limit for the saturable gain. Stable modelocked pulses are also obtained in the presence of both normal and anomalous group velocity dispersion when its magnitude is below a critical value. The stability limit for the saturable loss becomes less stringent when group velocity dispersion is simultaneously present. However, the stability limit for the saturable gain is not significantly affected. All these limits depend on the ratio of the SIT-induced gain and absorption to the linear loss. Realistic values for both the saturable nonlinearity and chromatic dispersion are within the range in which SIT modelocking is predicted to be stable.

OCIS codes: (140.5965) Semiconductor lasers, quantum cascade; (140.4050) Mode-locked lasers.

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#121880 - $15.00 USD Received 22 Dec 2009; revised 28 Jan 2010; accepted 21 Feb 2010; published 4 Mar 2010

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1. Introduction

Modelocking of quantum cascade lasers (QCLs) [1] is an active field of research. They are the most useful semiconductor sources that operate in the mid-IR, and modelocking these lasers opens a path to generating short pulses in the mid-IR from a semiconductor source. That in turn has a host of military, communications, medical, and environmental sensing applications. While the generation of 3 ps pulses has been achieved using active modelocking [2], it has proved difficult to generate sub-ps pulses from QCLs using conventional modelocking because of the inherently narrow linewidth and short gain recovery time T_1 compared to direct-bandgap semiconductor lasers [3]. The linewidth in QCLs is narrow compared to direct-bandgap semiconductor lasers because the lasing transition takes place between two subbands within the conduction band [4]. The subbands have approximately the same curvature, so that the transition frequency only has a weak dependence on the electron temperature. The gain recovery time of a conventional QCL is generally on the order of a ps due to very fast carrier transport by resonant tunneling and LO phonon relaxation [5, 6], while the round-trip time T_{rt} in a typical 3-mm-long cavity of conventional QCLs is around 50 ps. The condition $T_1 \ll T_{\text{rt}}$ makes it difficult to form a pulse, and the laser may become unstable when operated above its threshold due to the Risken-Nummedal-Graham-Haken instability [7].

Though the fast gain recovery time and the narrow linewidth of the conventional QCLs are not suitable for conventional passive modelocking [8–10], these conditions are ideal for using the self-induced transparency (SIT) effect [11–14]. The relatively narrow linewidth compared to direct-bandgap semiconductor lasers corresponds to a relatively large coherence time T_2, and the short recovery time implies that inversion in the gain medium will be restored during one round-trip of a short pulse. Recently, it has been theoretically demonstrated that QCLs can be modelocked using the SIT effect in their typical parameter range and that pulses on

#121880 - $15.00 USD Received 22 Dec 2009; revised 28 Jan 2010; accepted 21 Feb 2010; published 4 Mar 2010

the order of the coherence time T_2 can be generated [15–17]. For this purpose, QCLs must be grown with interleaved absorbing and gain periods. The absorbing periods stabilize the short pulses by absorbing continuous waves and shape the pulses as they propagate. These absorbing periods operate in a way that is analogous to a saturable absorber in a conventional passively modelocked laser.

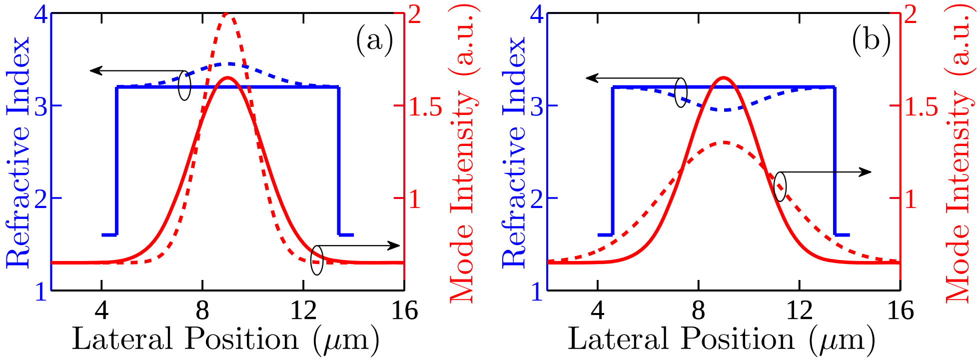

Fig. 1. Refractive index profile and resulting intensity distribution of the fundamental waveguide mode along the lateral direction of a conventional QCL waveguide in case of (a) saturable loss and (b) saturable gain. The intensity profile changes due to the strong index nonlinearity of the active region. (a) If the intensity increases, the index near the center of the waveguide increases and the mode is more tightly confined. (b) If the intensity decreases, the index near the center of the waveguide decreases and the mode is more weakly confined.

Paiella *et al.* [18] observed self-focusing due to a strong intensity-dependent refractive index in QCLs. In addition, Wang *et al.* [3] and Gordon *et al.* [19] found evidence for an intensity-dependent loss in QCLs. They found that the loss saturates when QCLs are operated with an input current that is above threshold. As the intensity of the light inside the laser increases, the intensity-dependent portion of the refractive index, n_2, adds to the intensity-independent portion of the refractive index, n_0. The total refractive index of the core increases. Therefore, the overlap of the lateral optical mode with the core increases, and the overlap with the lossy cladding decreases, as schematically shown in Fig. 1(a). As a result, the loss decreases. In practice, the change of the refractive index with intensity can be positive or negative, depending on the wavelength. When the sign of n_2 is negative, then the loss will increase because the overlap of the lateral optical mode with the lossy cladding will increase, as schematically shown in Fig. 1(b). Therefore, an optical pulse may experience saturable loss or saturable gain, depending on the wavelength of the laser transition.

Chromatic dispersion is also present in QCLs. Choi *et al.* [20] found for a particular QCL a dispersion coefficient $\beta_2 \sim -4.6$ ps^2/m at the 5 μm gain transition. Therefore, a pulse may broaden during propagation inside the laser structure.

Work to date on SIT modelocking of QCLs has not considered the intensity dependence of the refractive index when the pulse propagates in the laser structure or chromatic dispersion. In work to date, we have demonstrated analytically that SIT modelocking is possible under ideal conditions [15], and we have shown computationally that the modelocking is robust when the parameters of the Maxwell-Bloch equations vary from their ideal values [16]. We have also shown that SIT modelocking is robust when the geometric effects of bi-directional propagation and lumped mirror losses are considered [17]. Since the stability of SIT modelocking depends on the magnitude of the gain and absorption coefficients relative to loss, the modulation of

Received 22 Dec 2009; revised 28 Jan 2010; accepted 21 Feb 2010; published 4 Mar 2010

loss by the intensity will affect the stability of the SIT modelocking. Since SIT modelocking generates, in principle, pulses on the order of only 100 fs, chromatic dispersion, which spreads the pulses will also affect the stability. Therefore, it is critical to investigate the effects of the saturable nonlinearity and chromatic dispersion in order to determine the practicality of SIT modelocking.

In this work, we find that SIT modelocking is achieved in the presence of the saturable loss or the saturable gain, but only when the saturable loss and the saturable gain are below critical values. The limiting values of the saturable loss are significantly lower in magnitude than the limiting values of the saturable gain. The intensity and duration of the modelocked pulse depend on the amount of the saturable loss or saturable gain, in addition to the gain and absorption coefficients.

SIT modelocking is stable when the group velocity dispersion is normal or anomalous, in contrast to conventional passively modelocked systems that can be modeled by the complex Ginzburg-Landau equation [21]. In fact, as we will show, the pulse intensity as a function of time is the same regardless of the sign of the chromatic dispersion in SIT modelocking. The pulse intensity decreases and the pulse duration increases when the absolute value of the dispersion coefficient increases. Beyond a critical value that depends on the magnitude of the gain and absorption coefficients, the pulses become unstable. They either break up into multiple pulses due to the growth of continuous waves, or they damp away.

When saturable nonlinearity and group velocity dispersion are simultaneously present, as would be expected to occur in practice, the stability limit for the saturable loss increases, becoming less stringent, while the stability limit for the saturable gain remains almost unchanged. We will show that realistic values for the saturable nonlinearity and chromatic dispersion are within the range where SIT modelocked pulses are expected to be stable.

In this work, we do not consider the backward-propagating waves, which may interfere with the forward-propagating waves and create spatial hole burning. In Ref. 17, we showed that spatial hole burning is not an issue for SIT modelocking as long as continuous waves are suppressed, and continuous waves are always suppressed when SIT modelocking is stable.

In this work, we also do not consider the lumped mirror loss. The inclusion of the lumped mirror loss in the model does not change the results qualitatively, although there is some quantitative change [17]. The impact of the lumped mirror loss depends significantly on the cavity length. Quantum cascade lasers are generally 1–3 mm long. The impact of the lumped mirror loss on SIT modelocking will decrease as the cavity length increases. In order to focus on the effects of chromatic dispersion and saturable nonlinearity, we have added the mirror loss to the loss in the medium and distributed the total loss along the length of the cavity. However, if lumped mirror loss is included in the model, the realistic values for the saturable nonlinearity and group velocity dispersion will still be within the range where SIT modelocked pulses are expected to be stable. In Ref. 17, we found that due to lumped mirror loss, pulses decay with a smaller absorption coefficient for a given gain coefficient. Therefore, saturable loss, which sets the most stringent limits, will compensate the lumped mirror loss. The stability limits will increase, and SIT modelocking will be obtained with greater saturable loss.

The remainder of this paper is organized as follows: Sec. 2 presents the theoretical model of the pulse dynamics in QCLs that have interleaved gain and absorbing periods when saturable nonlinearity and group velocity dispersion are present. In Sec. 3, we show the effect of saturable loss or saturable gain on SIT modelocking. In Sec. 4, we show the effect of group velocity dispersion on SIT modelocking. In Sec. 5, we show the effect on SIT modelocking when both the saturable nonlinearity and group velocity dispersion are simultaneously present. Finally, in Sec. 6, we summarize the results and draw conclusions.

2. Theoretical model

We use the two-level approximation of the Maxwell-Bloch equations [22,23] to model the gain dynamics of a QCL. In SIT-modelocked QCL structures, the absorbing periods are interleaved with the gain periods. So, in the presence of saturable nonlinearity and group velocity dispersion, the Maxwell-Bloch equations can be written as

$$\frac{n}{c}\frac{\partial E}{\partial t} = -\frac{\partial E}{\partial z} - i\frac{kN_g\Gamma_g\mu_g}{2\varepsilon_0 n^2}\eta_g - i\frac{kN_a\Gamma_a\mu_a}{2\varepsilon_0 n^2}\eta_a - \frac{1}{2}l(|E|^2)E - i\frac{1}{2}\beta_2\frac{\partial^2 E}{\partial t^2}, \tag{1.a}$$

$$\frac{\partial \eta_g}{\partial t} = \frac{i\mu_g}{2\hbar}\Delta_g E - \frac{\eta_g}{T_{2g}}, \tag{1.b}$$

$$\frac{\partial \Delta_g}{\partial t} = \frac{i\mu_g}{\hbar}\eta_g E^* - \frac{i\mu_g}{\hbar}\eta_g^* E + \frac{\Delta_{g0}-\Delta_g}{T_{1g}}, \tag{1.c}$$

$$\frac{\partial \eta_a}{\partial t} = \frac{i\mu_a}{2\hbar}\Delta_a E - \frac{\eta_a}{T_{2a}}, \tag{1.d}$$

$$\frac{\partial \Delta_a}{\partial t} = \frac{i\mu_a}{\hbar}\eta_a E^* - \frac{i\mu_a}{\hbar}\eta_a^* E + \frac{\Delta_{a0}-\Delta_a}{T_{1a}}, \tag{1.e}$$

where the subscripts g and a in Eq. (1) refer to the gain and absorbing periods, respectively. The independent variables z and t denote length along the light-propagation axis of the QCL and time. The dependent variables E, η_g, Δ_g, η_a, and Δ_a denote the envelope of the electric field, the polarization and inversion in the gain medium, and the polarization and inversion in the absorbing medium. The parameters Δ_{g0} and Δ_{a0} denote the equilibrium inversion away from the modelocked pulse. The parameters μ_g and μ_a denote the dipole moments. The parameters N and Γ denote the electron density and the mode overlap factor. The parameters n, c, k, ε_0, and $\hbar$ denote the index of refraction, the speed of light, the wavenumber in the active region, the vacuum dielectric permittivity, and Planck's constant. The parameter β_2 is the dispersion coefficient. The parameter $l(|E|^2)$ denotes the loss that depends on the light intensity.

Equation (1) is the same as in [16] except that here we include the effect of chromatic dispersion, and we allow $l(|E|^2)$ to depend on the field intensity. We may write the intensity-dependent loss as [21]

$$l(|E|^2) = l_0 - \gamma|E|^2, \tag{2}$$

where l_0 is the linear loss and γ is the saturable nonlinearity coefficient. According to Eq. (2), the loss will saturate when $\gamma > 0$ and the gain will saturate when $\gamma < 0$. The sign of γ depends directly on the sign of n_2, which in turn depends on the wavelength of the laser transition [23]. Gordon *et al.* [19] found that γ depends sensitively on the width of the QCL active region. As the width of the active region increases, the parameter γ decreases. Gordon *et al.* found that γ decreases approximately by a factor of two when the width of the active region increases from 3 μm to 7.5 μm.

The performance and the stability of the SIT modelocking of QCLs depend on the gain and absorption coefficients from the gain and absorbing periods, respectively. The gain per unit length (g) and the absorption per unit length (a) are given by [16]

$$g = \frac{kN_g\Gamma_g\mu_g^2 T_{2g}}{2\varepsilon_0 n^2\hbar}, \qquad a = \frac{kN_a\Gamma_a\mu_a^2 T_{2a}}{2\varepsilon_0 n^2\hbar}. \tag{3}$$

The stability of SIT modelocking does not depend on the absolute values of the gain and absorption coefficients, but rather on their values relative to the loss in the system. Therefore, it is useful to normalize the gain and absorption coefficients relative to the linear loss, i.e., $\bar{g} = g/l_0$ and $\bar{a} = a/l_0$. In this work, we have set $T_{2g} = T_{2a} = 100$ fs and $T_{1g} = T_{1a} = 1$ ps. We have set

#121880 - $15.00 USD Received 22 Dec 2009; revised 28 Jan 2010; accepted 21 Feb 2010; published 4 Mar 2010

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